Dimension Reduction of Solid Models by Mid-Surface Generation

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Abstract

Recently, feature-based solid modeling systems have been widely used in product design. However, for engineering analysis of a product model, an abstracted CAD model composed of mid-surfaces is desirable for conditions in which the abstracted model does not affect analysis result seriously. To meet this requirement, a variety of solid abstraction methods such as MAT (medial axis transformation) have been proposed to provide an abstracted CAE model from a solid design model. The algorithm of the MAT approach can be applied to any complicated solid model. However, additional work to trim and extend some parts of the result is required to obtain a practically useful CAE model because the inscribed sphere used in the MAT method generates insufficient surfaces with branches. On the other hand, the mid-surface abstraction approach supports a practical method for generating a two-dimensional abstracted model, even though it has difficulties in creating a mid-surface from some complicated parts.

In this paper, we propose a dimension reduction approach on solid models based on the mid-surface abstraction approach. This approach simplifies the solid model by abbreviating or removing trivial features first such as the fillet, mounting, or protrusion. The geometry of each face is replaced with mid-patches from the simplified model, and then unnecessary topological entities are deleted to generate a clean abstracted model. Also, additional work, such as extending and stitching mid-patches, completes the generation of a mid-surface model from the patches.

Key words: dimension reduction, mid-surface, geometry replacement

1. Introduction

Commercial feature-based solid modeling systems with powerful design and analysis sub-functions are widely used nowadays in most industrial fields for product design. The CAD system helps a designer create and modify detailed and specified design of a product. Also the featured design file can be used for the analysis of a product's physical properties, for dynamic analysis, and manufacturing analysis. These analysis are executed within the CAD software itself or within the corresponding analysis tool which works with the converted design file obtained

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Fig. 1. (a) Solid model, (b) Mid-surface model, and (c) Mesh model for analysis.

from feature data. However, dimensionally abstracted model is occasionally preferred over a full solid model within the analysis process. In this case, the detailed model needs to be simplified to a surface model that does not contain complicated topology information for efficient analysis.

In the vehicle industry, a design model of a vehicle part is used for virtual car crash tests using a simulation package. Generally, the simulation process does not need detailed feature data of each part, and a solid model of the part, that shown in Fig. 1. (a), is converted to a simple geometrical model for analysis. A mid-surface model, such as that shown in Fig. 1. (b), is usually generated and used because of its suitability for this purpose. Fig. 2 shows this operation conducted in two steps. The first step removes the detailed and unnecessary features which are not important factors in the analysis of the model. Then, a mid-surface is created between the outer and inner surfaces of the part model by using free surface functions supported in the feature-based modeling system or analysis tool. However, creating a mid-surface model is a very tedious stage to handle manually; therefore, it becomes a bottleneck, which further prolongs the whole analysis process.

This paper suggests a practical solution of mid-surface generation, which can be applied in the post design process, to shorten the time lag of converting a model type, and implements this solution under the CATIA V5 CAA [1] environment. After the discussion of related works in the following chapter, this paper suggests a general approach for dimensional reduction in chapter 3. The specified algorithm of simplifying and generating a mid-surface is presented in chapters 4, 5. The examples and case study of the research are provided in chapter 6, and the last chapter concludes and discusses future work plans.
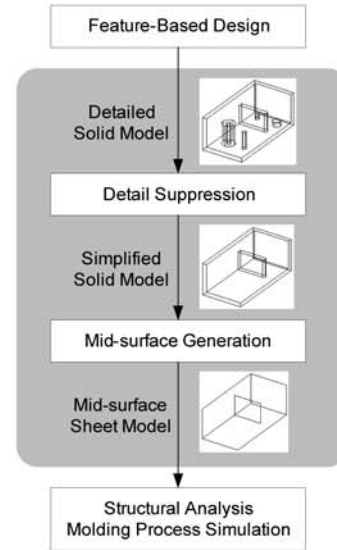

Fig. 2. Solid simplification and thinning processor

2. Related Work

As a strategy of solid simplification, the generation of design features stored in the feature tree is has become very common. One of the approaches for this purpose is the "smooth-out" method, which removes small design features with loops of convex edges [2]. Zhu [3] suggested an algorithm to identify fillet/round efficiently and to remove them in order to simplify the target model. These ideas are combined and a simplification strategy was suggested in [4] as well. Lee [5] suggested a concept of effective volume in which the design features are rearranged, regardless of the fact that the feature is may be additive or subtractive. A variety of approaches for abstraction and dimensional reduction of solid to create an analysis model has been suggested. Especially, the Medial Axis Transform (MAT) and Mid-surface abstraction
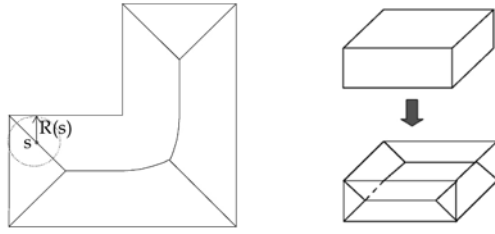
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Fig. 3. Medial Axis Transform.

have been proposed as dimensional reduction methods of solid models. Both approaches indicate an algorithm that abstracts the geometry property of a solid model, and converts a full three-dimensional solid to a surface model. Medial Axis (MA, or skeleton) was proposed for the first time by Blum [6, 7] to present a physical approach of the biological geometry and shape. Blum defined Medial Axis Function (MAF) as a methodology to generate MA by contour descriptions. This idea is remarkable in terms of the fact that a shape has a unique MA in the processing and vice versa. It can reduce the dimensions of a shape while conserving its geometry information as well. The concept evolved to MAT, which creates a medial axis through a maximal inscribed disk moving on a shape, and defines MA and the radius function. Medial axis (medial surface in 3D) is determined as the locus of the center of a maximal inscribed disk (a sphere in 3D). MAT is adjustable to diverse shapes of a model even

though the model geometry may be complicated. It is, therefore, an approach for mesh generation [8, 9, 10], and decomposition of a solid model [11, 12, 13]. However, the medial surface retrieved by this method has little and insufficient branch edges or faces to be trimmed. These branches are the resultant shapes of an inscribed sphere. Also, the medial surface is smaller than the outer face, so an extension operation is needed to complete the process.

Mid-surface abstraction was proposed by Rezayat [14] to solve problems of insufficient entities in MAT. A ray casted from an arbitrary face in the direction of thickness finds a pair of faces, and an adjacent graph is drawn at the base of the topology information of the model. Mid-surfaces are generated by 2D Boolean operation and geometric interpolation, and they are sewn according to the adjacent graph. This method offers a practical and fast approach for generating a mid-surface model, and it is able to reduce additional healing operations to complete a surface model. It does, however, have limited application; it is difficult to apply to a complicated solid model

3. Overall Process of Our Approach

We propose a method, similar to the mid-surface abstraction method, for generating a mid-surface model [14]. Because of its practicability and efficiency, it can be utilized for simple cases. Even though this method is
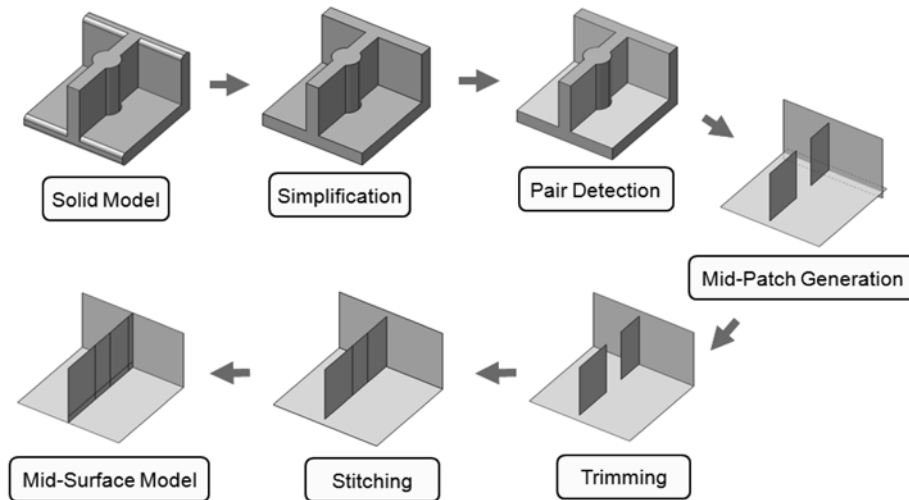

Fig. 4. Mid-surface model generation work flow

difficult to apply to a complex solid model, it can cover most subject models because a car part model usually consists of simple aggregated features.

The dimensional reduction process of a solid model we present consists of two main steps. The process is similar to practical methods used in the industry. Fig. 4 is a process flow of our approach used to create a mid-surface model.

The resultant mid-surface model is used for mesh generation in the analysis package, and therefore, it may not conserve the topology information of an untouched solid mode. From this point of view, this paper suggests the abstraction method as dealing with geometrical information primarily, and the pair information and/or adjacency are/is considered as topological information.

The first step is the simplification of the detailed solid model. This is a pre-process operation for generating the mid-surface model. Before the generation of the mid-surface model, the original detailed model should be simplified. Detailed and specified figures on a main part model make the analysis process more difficult and complicated because they are usually created for supportive reasons or subsidiary purposes.

Generation of mid-surface model is the next. The algorithm is divided into four steps as follows.
(1) Face pair detection
(2) Mid-patch generation
(3) Trimming
(4) Extending and stitching

To define mid-surface of a solid model, a pair of faces, which acts as a reference in the generation of the mid-surface, should be determined first. Within the face pair detection module, the face pairs, which will be substituted as the mid-surface in later steps, are searched in the simplified model. At the same time, non-pair faces in the model are classified as a useless entity in the mid-surface model generation. After pair detection, mid-patches are generated from the paired faces. When the faces of different areas are a pair, a mid-patch is created from the larger face by offsetting the larger one because this allows the reduction of additional jobs such as extending or patch stitching. In this case, some of the mid-patches are intersected, and the model has a small branch that needs to be trimmed. This branch is cut by the trimming

operation to be an interim clean model. The mid-patches generated in the previous step are separated from each other when they originate from the face pair and cannot complete the geometrical property. The separated patches need to be extended and connected to each other to become the resultant mid-surface model. The margins between the patches are extended by the extrude or extrapolate function of CAA and stitched from one to other, as shown in the Fig. 4.

4. Simplification of Detailed Model

Subsidiary features such as fillets and bosses usually exist on the general solid model for supportive purposes or assembly of each part. These features are essential in the full solid part model but they are not needed for the finite element analysis. Moreover, they have a negative effect on the level of complexity or computational time. At the same time, they cause complex or unwanted results, which need manual healing process before they are delivered to the analysis process. As a result, the user cannot get an exact mid-surface model from the detailed solid model because of these subsidiary features. Therefore, a detailed model should be operated in the process of simplification before the generation of the mid-surface as the pre-process work. The subsidiary features to be removed are defined as "Small Features" in this paper.

Two methods for removing the small features exist. One uses the feature tree information of the model, and the other uses only the geometric information of model when the feature tree information cannot be retrieved. The latter method is generally used for model simplification of model regardless of the availability of feature tree information, because it can be adjusted to a diversity of models. Also, feature tree or feature creation information is not always retrieved from feature-based modeling systems. Nevertheless, this paper applies feature create information, because CATIA V5 offers powerful functions to deal with creation information or feature tree, and it gives practical operation efficiency. Fig. 5 indicates the simplification process of a solid model using feature tree

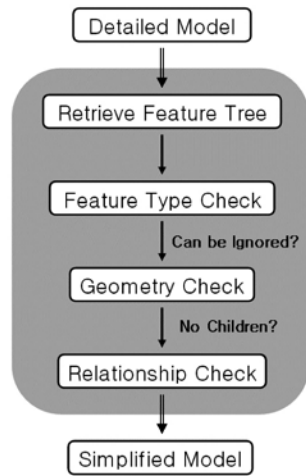A feature tree of a model in CATIA V5 contains lots of basic information on the model

Fig. 5. Algorithm for simplification

such as parameters, references, supports and sketches. Also, information about the procedure of modeling, geometry and type of features, and relationship of features are offered by the feature tree. To use the information referred above for the simplification procedure, the feature tree of a solid model and its information should be retrieved first. While retrieving the feature tree, the system checks the type of a feature. The features are of different types like pad, pocket, edge fillet, and chamber in the modeler. Even though the shape and geometry may look the same, sometimes the type may be different, depending on the way how the feature is modeled by designer.
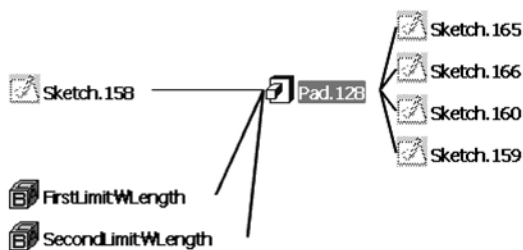

Fig. 6. 'Parent-Child' Relationship

After type checking, different detailed functions can be applied to the model according to the type of a feature. One of the most common features revealed in a detailed model is fillet or rounding. This feature is a mandatory element to be dealt with in solid simplification. When a fillet feature is found, the module retrieves specific information about the geometry of a fillet feature. The information of a fillet can be used to determine whether the features can be ignored for the analysis or not. For example, although a feature is determined to be a fillet feature, the fillet information should be reserved if the radius of the fillet is big enough such that it cannot be ignored in the analysis. The necessity of the feature is usually determined by the user's input.

The next step is checking the dependency relationship of the feature to be eliminated to other neighboring features. If feature A refers to another feature called feature B when feature A was created by the designer, feature A cannot be removed independently because deletion would affect the creation information of the original model. This means if we remove feature A, then feature B cannot define itself and an error would result. Relationship check of a feature should be done before deletion. In CATIA V5, the dependency relationship among features is defined as the "Parent/Children" relationship. For example, A is a parent and B becomes the children. To remove the parent element, isolation operation has to be applied to the parent to cut the relationship .of the parent to its children. In Fig. 6, Pad.128 is a parent, and Skecth.158, Sketch.160, Sketch.159, Sketch.165, and Sketch.166 are the children of Pad.128. To remove the feature of Pad. 128, the children entities of Sketches should be cut by the isolation operation..

After the relationship check has been completed, the small features can be eliminated. If all the necessary small features are selected properly, the removing operation is executed to the target small features to conclude the simplification, as shown in Fig. 7.
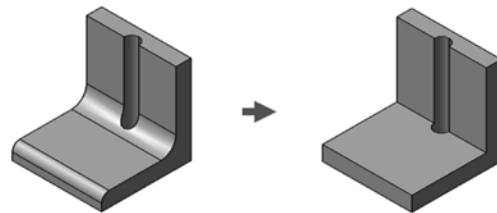

Fig. 7. Removing fillet from a part model

5. Mid-Surface Generation

The simplified solid model whose trivial features are removed is now ready to move forward to the mid-surface generation process. This process consists of 4 minor steps. The first step finds and detects face pairs, which can be the foundation in the generation of mid-patches. The pairs are determined under certain conditions: parallel and/or symmetry of two faces, distance between them, and overlapping test. Mid-patches are created at the base of the face pairs detected in the preceding process. In this step, pairs are replaced by newly generated mid-patches, which are incomplete mid-surface models. To avoid a complex operation, mid-patch is offset and newly created from the larger face of the pair when the areas of the faces are different. This reduces the additional and trivial operation of extending one patch to the other. Some regions intersect with small branch faces which are results of offsetting the larger faces of a pair. The intermediate model is processed with trimming operation on the branch faces. In the last step, the patches are extended and stitched between separated patches to complete the surface model.

5.1. Face Pair Detection

To create a mid-surface from a solid model, target faces to be replaced to mid-surfaces should be determined first. Mid-surface is generated based on the following principles: preservation of the volume of the solid model and reflection of the form of the part [14]. If a pair is determined, a mid-surface can be created based on the relation between the two faces. Basically two similar and opposite faces at a small distance can be candidates for a face pair. Maximum thickness (t_{max}) and maximum draft angle (Θ_{max}) of the part model are required as user input parameters to determine the pair. Face pairs are classified based on the following conditions:

(1) In the case of normal extrusion faces, directions of the normal vectors (N1, N2) of the two faces (F1, F2) should be same or opposite. In short, they must be parallel, such as Fig. 8 (a).

$$N_1 \cdot N_2 = \pm 1 \qquad (1)$$

(2) In the case of draft faces, the angle between two faces is smaller than doubled maximum draft angle as shown in Fig. 8 (b).

$$180^o - \text{angle}(N_1, N_2) < 2\Theta_{max} \qquad (2)$$

(3) The distance between two faces should be smaller than the maximal thickness

$$d < t_{max} \qquad (3)$$

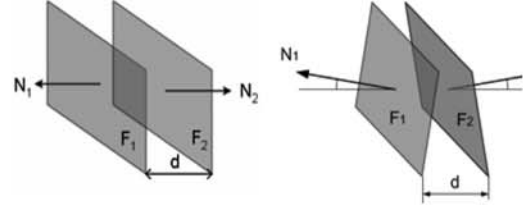(4) When one face is projected onto the other, they must overlapped

Fig. 8. Parallel check.

(a) Parallel Pair, (b) Draft Pair

The face pair detection module has the following algorithm to narrow down the pair candidates under the four pair conditions:
(1) Parallel check
(2) Distance check
(3) Overlap test
(4) Pair decision

In this algorithm, parallel faces are two faces that are exactly parallel (condition 1), or they are in a similar pattern in draft (condition 2). All the faces existing in a part model are searched and only the pairs which correspond with condition 1 or condition 2 are determined as face pairs. Fig. 9 (a) shows the faces that are not determined as a pair candidate because the relationship between the two faces violate condition 1 or condition 2. Even though two faces are parallel in a part model, they cannot be selected as a pair if the distance between them is larger than the maximal thickness of a part model. Condition 3 is applied as a second filter to determine a face pair. Faces of Fig. 9 (b) are determined as parallel at parallel check, but they cannot be a pair because the distance between them is greater than the maximal thickness. If a model has many faces as pair such as model of Fig. 9., the module may select the adjacent faces , as shown in Fig. 9 (c) as a pair. To prevent this error, the overlap test is executed by the projection of one face to the other to determine whether or not they intersect
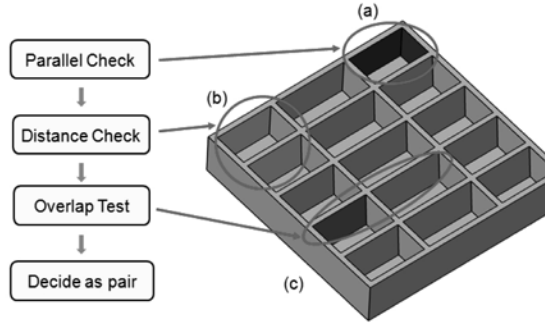
Fig. 9. Face Pair Detection. (a) Parallel Check,

(b) Distance Check, (c) Overlap Test

Two faces that pass these three steps can be a pair for the generation of a mid-surface. Sometime, a pair face may be repeated as a pair of other pair. In this algorithm, we defined a pair which does not mean only two matching faces. This means that one face can be a pair to several groups of faces if they satisfy the face pair conditions. The one large face, shown on the bottom of Fig. 9, and 12 small faces matching to the large one is defined a pair as well.
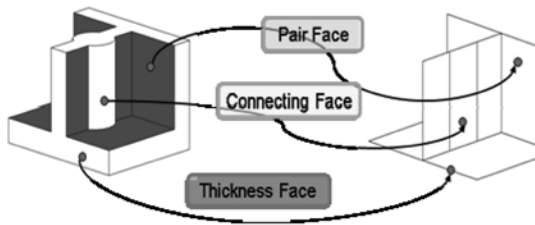
Fig. 10. Face Relationship between

(a) Solid Model, (b) Surface Model

While determining the face pairs, the algorithm finds non-pair faces or useless faces for mid-surface generation. They are categorized to connecting faces or thickness faces according to their topology information. Only pair faces are used and replaced to a mid-surface by the dimensional reduction process. Fig. 10 and Table 1 indicate face categories and their relationship between solid and surface models.

Table 1. Transformation from solid to surface model

Type	Remains in result	Transformation
Pair Face	O	Mid-surface patch
Thickness Face	X	Edge
Connecting Face	X	Edge or Merged Adjacent Face
	O	Remains as Face

5.2. Mid-patch Generation

After classifying all surfaces of a solid model into pairs and non-pairs, the main operation of mid-surface generation is started. The mid-patches made in this module are the bases of the mid-surface. Creating appropriate mid-patches helps to complete mid-surface model with great efficiency. Mid-patch generation method is different according to the type of the face pair. The types are the parallel pair and the draft pair. These types were suggested in chapter 5.1. to determine the face pair. They are applied in this step as well. To generate mid-patches, offsetting and projecting are used. If a pair is determined as a parallel pair, one face is offset to create the mid-patch. In the case of draft pair, a face is projected to the mid-plane of the two faces, and the projected face becomes the mid-patch.

Solid of a part consists of many faces, and of course, it may have several parallel pairs especially in the case of a planar solid model, as shown in Fig. 9. Parallel face pairs were detected in the previous step of determining face pairs; therefore, generating a mid-patch can be executed very simply. If a pair is given, the module refers to the distance between the two faces decided as the input face pair, which was already calculated in the preceding process, to obtain a thickness. Then, an arbitrary face between the two faces of the pair is selected as the reference face. This face is offset by the amount of half thickness or distance between the two faces in the direction of opposite face with the normal vector of the reference face.

However, many models are required to select the proper face as the reference face because this may reduce complexity the whole process and complete the work faster and more efficiently. Here, a tip for selecting the appropriate face as the reference face is suggested: check the area of each face in a

pair, and compare the areas of the faces. The module uses the larger face as the reference. Fig. 11 indicates the difference between two methodologies: one is to select and offset the smaller face, and the other is to select the larger face when the faces of a pair have different areas. If mid-patches are generated from the smaller face, number of patches will increase because more mid-patches are needed to connect the patches to each other to complete the mid-surface model, and as a result, computation becomes much more complicated in the geometric progression for the later process. Hence, we can shorten the whole computational time and complexity by selecting the larger face as the reference.
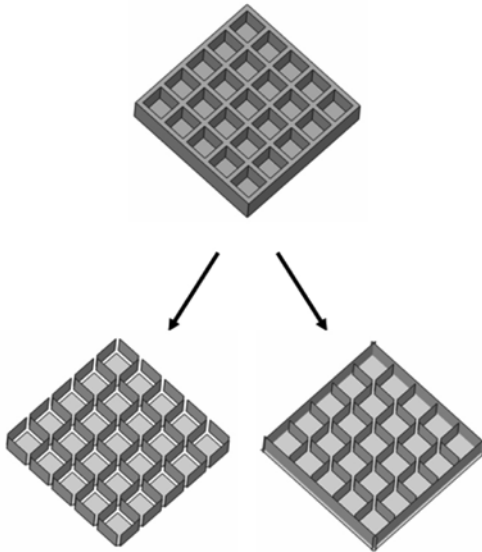
Fig. 11. Difference between two mid-patch generation

methods

For the case of a draft pair, mid-patches are more difficult to generate than for the case of a parallel pair. Because the distance between two faces is not uniform, the offset from the drafted face produces a result different from the parallel pair. Therefore, the module generates mid-patch by an indirect and complicated approach, unlike the case of a parallel face pair. First, the pairs of vertices between the two faces are defined. At the middle point of the pair of vertices, a new point (mid-point of two vertices) is created. If we assume that two faces of a draft pair are symmetric, a plane can be create by generating 3 mid-points of vertices pairs, which become the mid-plane of two drafted

faces. Now one of the pair faces is projected onto the mid-plane in the direction of the mid-plane's normal to generate a mid-patch. Fig. 12 shows the process of mid-patch generation for a draft pair.
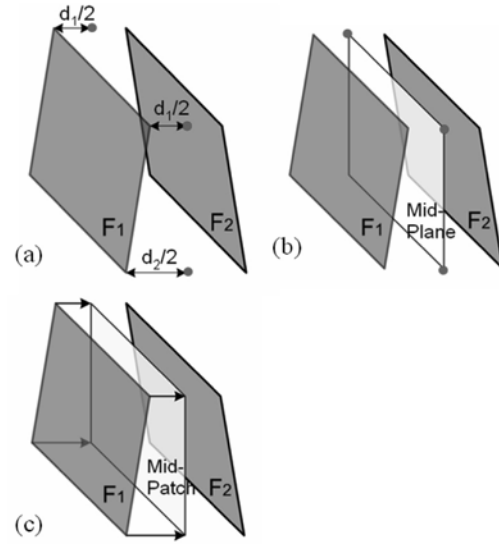

Fig. 12. Process of mid-patch generation for draft pair,

(a) create mid-point, (b) create mid-plane,

(c) generate mid-patch by projection

5.3. Trimming

Because the bigger face of a pair is selected as the base of the offset when generating mid-patches, the model shows mid-patches intersecting each other. These small parts may confuse the process of extending and stitching. Therefore, small patches should be eliminated by the trimming operation to achieve more efficient procedures. Also this operation can decrease additional works, which might result from the small patches in the latter process.

Trimming operation is executed as follows. Parts (or patches) having intersections are detected first, and the unnecessary parts are removed by the trimming operation. Fig. 13 shows the parts with intersections resulting from mid-patch generation, and the results after executing the trimming operation.

5.4. Extending and Stitching

All the mid-patches generated in 5.2 exist separately, except the faces trimmed in 5.3 because
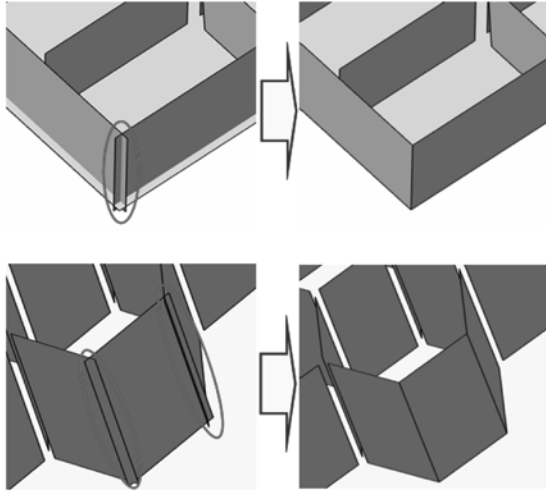
Fig. 13. Intersected patches and result of trimming operation

they were created by using offsetting or projection of a target face. The resultant model of mid-patch generation and trimming operation has only a rough shape of the model, but was not completed as a "mid-surface model" to be applied for analysis. For this reason, extending and stitching operation was defined for the completion of the mid-surface model. This process connects the non-continuous mid-patches created in chapter 5.2 and the surface groups formed in chapter 5.3 and makes one complete mid-surface model. The two operations of expanding and trimming were used for minor functions of this module, as shown through Fig. 14.
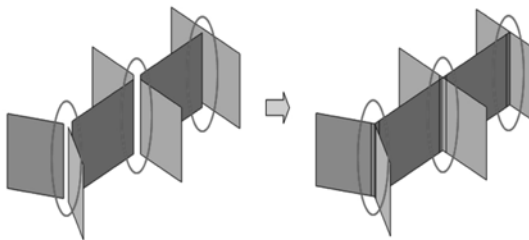
Fig. 14. Extending and Stitching Operation

Patch connecting type is classified into three categories according to their cross and connection shape. The first type is named the "in plane type"; two adjacent patches are placed on the same plane. If they are connected by the extending and stitching operation, they will compose one continuous surface, as shown in Fig. 15. (a). The other type is named the "T-joint type", one patch extended in certain direction touches the other patch, as shown in Fig. 15. (b). The last type is the "L-joint type"; neither of the two patches is not touched if only one patch is extended. This means they are touched only if both patches are extended like Fig. 15. (c). Most mid-patch models consist of these three types, and these types cover 70~80 percent of the possible connecting types in a one part model.

To distinguish the connecting type, the normal of each patch and the distance between the two patches are calculated to find patch-patch pairs and edge-edge pairs coincident to the patch-patch pairs. For the first time, the normal of each patch is checked and is compared to a normal of the neighboring patch. If they are found to be parallel and their distance from each other is smaller than the maximal thickness used in 5.1, they are determined as the "In-Plane" connecting type. If two patches are not parallel, they can be categorized as the "T-joint type" or "L-joint type". To determine the joint type, one patch is extended from one edge of the edge-edge pair. If the extended patch touches the other, it becomes the T-joint type, and if not, it is the L-joint type.

Extending and stitching strategy varies according to the connection type. In the case of "In-Plane" type, one edge of the edge-edge pair is extended to the other edge of the pair using the 'up-to' option of the extrapolate function. A similar method is applied to the T-joint type, although its connection and adjacent information is different from "In-Plane" type. One of differences of extending on T-joint from doing on "In-Plane" extending is that T-joint has an edge-patch pair instead of an edge-edge pair. The edge of an edge-patch pair is extended to the target patch in the option of 'up-to'. For the L-joint, one edge of the edge-edge pair is extended by maximal thickness. In the same manner, the other edge of the edge-edge pair is extended by the same amount. Two extended patches have an intersection, as shown in Fig. 15. (c). The overextending patches are trimmed from each other and the process is completed.
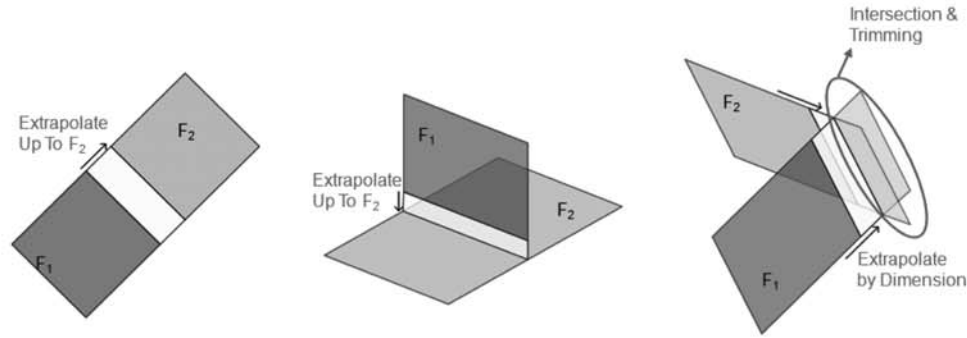
Fig. 15. Extending and Stitching Operation on

(a) "In-Plane" Type, (b) T-joint Type, and (c) L-joint Type

6. Case Study

The methodology was implemented under Visual C++ environment with CATIA V5 CAA, which is an application program interface for the CATIA modeler. The implemented program was tested with several simple models. The feature tree of CATIA was constructed with a sub-structure as the process was executed to allow the user to understand the procedure and manage additional operations when needed. Fig. 16 (a) is the feature tree structure implemented by CAA. Each of the branches contains entities, which are created at each step, as shown in Fig. 16 (b).

Fig. 17 is an example of dimensional reduction flow from face pair detection to completion of mid-surface model generation. Fig. 18 is another example of this process

7. Conclusion

A Mid-surface abstraction method, which can be adapted to a simple solid model, was proposed in this paper. This method is based on simple and clear ideas in which the fundamental geometrical properties are accessed, and suggests a fast and efficient methodology with practical approaches. At the same time, a simplification method provides a positive answer on dealing with a complicated model, and it leaves a possibility for evolution for adaptation to more complex geometries. However, its extensibility to adjust to diverse object models is inferior to the Medial Axis

Transformation. This means that a number of special cases will be difficult to implement with general approaches. It also means that this method has a limitation on dealing with all cases: some cases would require a range of manual works. In addition, the face pairing method also has weak-points. This method is limited to modeling where feature tree is provided, and all small features cannot be simplified.
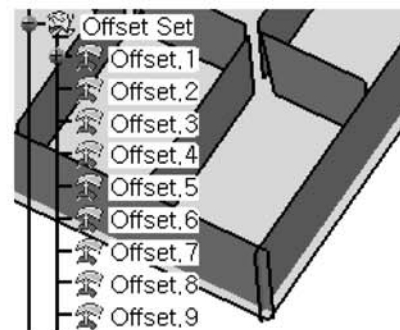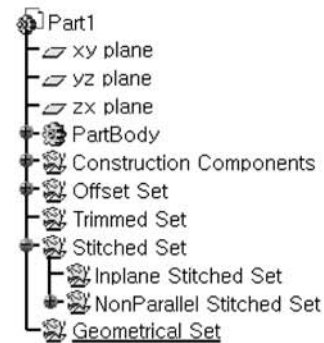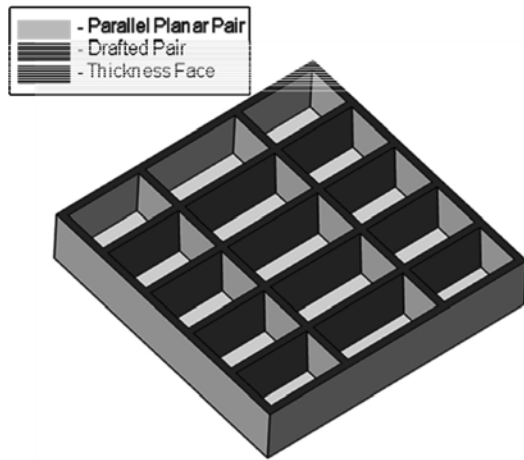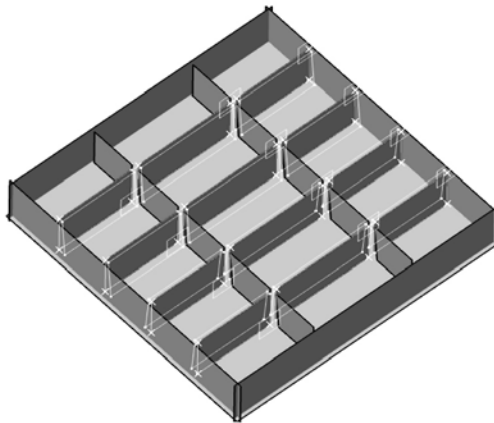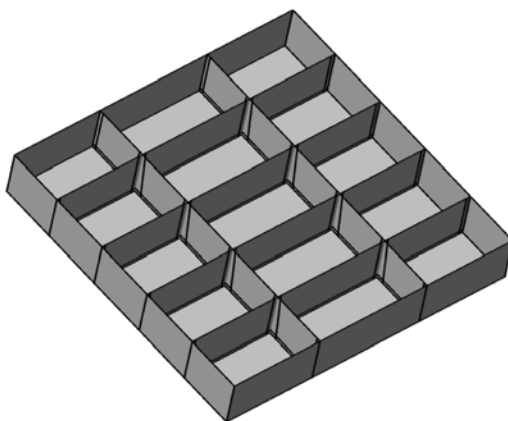

Fig. 16. Feature Tree Structure, (a) Tree structure,

(b) Entities generated in offset operation
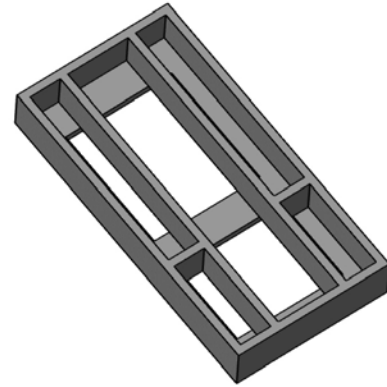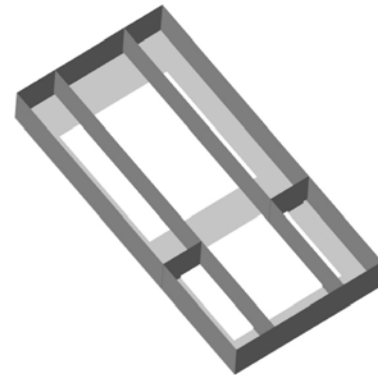
250

(a)

(b)

(c)

Fig. 17. Dimensional reduction process

to mid-surface model

(a)

(b)

Fig. 18. (a) Solid model, (b) Mid-surface model

For the future plan, the implementation needs to consider many exception or special cases. Because all possibilities cannot be covered, frequent occurred cases are solved automatically or semi-automatically. From this point of view, the algorithm has to be evolved for application to more diverse geometries. Also, extension and projection methods to a curved surface are to be researched. For example, inplanar pair extension method should be enlarged to encompass the scope of tangential continuity between two faces. Simplification method can be modified for volume decomposition or rearrangement of the created information to improve the abilities of the process.

Acknowledgement

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