# Multi-Resolution Modelling for Feature-Based Solid Models Using the Effective Volumes of Features

Sang Hun Lee

Kookmin University , shlee@kookmin.ac.kr

## ABSTRACT

Recently, three-dimensional CAD systems based on feature-based solid modelling techniques have been widely used for product design. However, when part models associated with features are used in various downstream applications, simplified models at various levels of detail (LODs) are frequently more desirable than the full details of the parts. One challenge is to generate valid models at various LODs after an arbitrary rearrangement of features using to a certain LOD criterion, because composite Boolean operations consisting of union and subtraction are not commutative. This paper proposes an algorithm for feature-based multi-resolution modelling based on the effective volumes of features. This algorithm guarantees the same resulting shape and the reasonable intermediate LOD models for an arbitrary rearrangement of the features, regardless of whether feature types are additive or subtractive. This characteristic enables various LOD criteria to be used for a wide range of applications.

**Keywords:** Multi-resolution; Level of detail; Feature; Solid model; Non-manifold; Merged set.

## 1. INTRODUCTION

Recent three-dimensional CAD systems based on solid modelling functionality have been widely used for product design in manufacturing companies. This is possible, as product-modelling environments have been greatly improved by the introduction of feature technologies associated with parametric or variational modelling techniques. A feature can be defined as a physical constituent of a part that has engineering significance and is mappable to a generic shape [21]. When product models associated with features are used in various downstream applications, however, simplified models at various levels of detail (LODs) are frequently more desirable and useful than the full detailed model. For example:

- In engineering analysis, most cases require a simplified model that idealizes the part geometry depending upon the analysis tools, rather than the full details of the part [1, 2]. This involves suppressing the detailed features to produce a simplified model. The LOD of the part may vary according to engineer's intent, analysis method, accuracy of results, system performance, or other factors.
- In the distributed design environment, the efficient transmission of solid models over the network is necessary for efficient collaborative design and manufacturing [3, 11, 16, 23]. However, this is difficult because the B-rep is very complex and network bandwidth is often limited. To overcome these limitations, it is necessary to transmit solid models incrementally and to share the model at adequate LODs depending on the engineering tasks. As the design and simplification of the part is performed by feature, feature-based multi-resolution modelling and streaming is necessary in this domain.
- In virtual prototyping, a digital mock-up (DMU) or virtual prototype, which is a complete assembly consisting of three-dimensional geometric models of individual parts, is built for visualization of the assembly and as a verification of the feasibility of an assembling operation. In virtual manufacturing, all facilities in a factory, such as robots, conveyors, fixtures, buffers, docks, and work cells, are modelled and used for the simulation and visualization of a virtual factory [12]. As digital mock-ups and virtual factories contain a huge amount of geometric data, LOD techniques are essential to perform rendering, collision detection, and various engineering analyses and simulations. In particular, when parts are simplified, a method to suppress detailed features such as holes and fillets according to a given LOD is recommended to reduce data storage while preserving the global part shape and simulation

accuracy [5, 10]. For fast rendering, any conventional mesh simplification method [6, 20] also can be applied to the simplified feature model [11].

Consequently, the requirements of multi-resolution models of a solid model, which represent an object at multiple levels of feature detail, are increasing for engineering tasks such as analysis, network-based collaborative design, and virtual prototyping and manufacturing.

To meet these requirements, several researchers have recently investigated multi-resolution modelling techniques for feature-based solid models [5, 11, 14]. This research has focused on several topics:

- Topological frameworks for representing multi-resolution solid models: Two approaches have been identified, using either conventional solid data structures [5] or non-manifold cellular structures [11, 14]. In the conventional solid data structure approach [5], the multi-resolution model is represented by a feature tree in which features are rearranged according to a criterion of LOD. If a simplified model at a certain LOD is required, the system prunes the branches of the feature tree and performs boundary evaluation to obtain a corresponding solid model. This method has an advantage because it may be implemented in current commercial 3D CAD systems, as they share the same data structures. However, it requires much computation time to evaluate boundaries, as the transition from the current LOD m to the destination LOD n requires $|m-n|$ Boolean operations (which are the most time-consuming operations used in solid modelling). To solve this problem, the non-manifold topological (NMT) model of a cellular structure was introduced as the topological framework of a multi-resolution model [14]. In this method, all features are merged into an NMT cellular model first, and then, if a LOD is given, the topological entities that constitute the model at the required LOD are selected and displayed. Since the boundary information of all of the features is stored in the NMT cellular model, boundary evaluation is performed for a solid model at the LOD. As a result, a model at a given LOD can be provided more quickly than if the solid-based approach is used.
- Criteria of LOD: These are dependent upon the intended applications of feature-based multi-resolution solid modelling. Currently, for the purpose of rendering and streaming solid models, the volume of the subtractive feature has been suggested as a criterion of LOD [5, 11]. In this method, the lowest resolution model is made by uniting all of the additive features, and then higher resolution models are generated by applying subtractive features successively in the descending order of volumes. However, this method has several limitations. First, if a part is modelled using only additive features, it can only have a single LOD. Second, this method assumes that subtractive features offer more detail than additive features; however, this may not be the case. Another criterion of LOD is the volume of the feature, regardless of the feature type [14, 15]. As there is no limitation on feature types, it may be adopted across a wider range of applications. More suggested criteria of LOD are expected as new application areas are discovered.
- Generation of valid models after rearrangement of features: Research has been conducted to generate valid models at various LODs after the feature rearrangement, based on the LOD criterion. In general, if features are rearranged, the resulting shape is different from the original because union and subtraction Boolean operations are not commutative. To avoid this problem, the proposed algorithm currently only uses a feature arrangement method based on the volumes of the subtractive features [5, 11], as mentioned above. However, to apply multi-resolution solid modelling to a wide range of application areas, the final result must be the same as the original shape, and the intermediate LODs of models must have reasonable shape, even though features are rearranged arbitrarily regardless of whether the feature is additive or subtractive. There is currently no definite solution to this problem.

In this paper, a solution to this feature rearrangement problem is proposed that introduces the new concept of the effective volume of the feature. The effective volume of a feature is defined as the actual volume of the feature in the rearranged feature tree, when used as a tool body for the Boolean operation. When arranged in the order of feature creation, the effective volume of each feature is defined as the entire volume of the feature. However, after feature rearrangement, the effective volume of a feature can be reduced to a fraction of the original volume. This paper describes a method to identify the effective volume and a mathematical proof of the method's correctness. By introducing the concept of effective volume, an arbitrary rearrangement of features becomes possible and arbitrary LOD criteria may be selected to suit various applications. In addition, a non-manifold model, known as a merged set [7], is introduced to represent a multi-resolution model. Because the merged set contains all boundary information about features, models at various LODs may be extracted quickly without performing any boundary evaluation.

The remainder of the paper is organized as follows. Section 2 describes the multi-resolution representation for feature-based solid models, which includes a merged set and an ordered list of the multi-resolution features whose attributes store all necessary information for multi-resolution modelling. Section 3 introduces the concept of the effective volume of the feature and the method used to identify the effective volume of a rearranged feature. Section 4 describes the algorithms used to create and store a multi-resolution model, and to extract the boundary representation of a solid model from the model at a given LOD. Section 5 presents several conventional and newly proposed LOD criteria. Some conclusions and future work are discussed in Section 6.

## 2. MULTI-RESOLUTION REPRESENTATION FOR FEATURE-BASED SOLID MODELS

To facilitate multi-resolution modelling for feature-based solid models, the proposed system stores and manipulates two types of data: the non-manifold cellular model, called a merged set, which contains all geometric data for the multi-resolution model; and a list of multi-resolution features, whose attributes contain all necessary information to build a multi-resolution solid model and extract LOD models from it. Here, a LOD model means a solid model at a specific LOD. Figure 1 shows a feature tree to create an example solid model by applying five form features. This model will be used throughout this paper to explore the proposed multi-resolution modelling method for feature-based solid models.

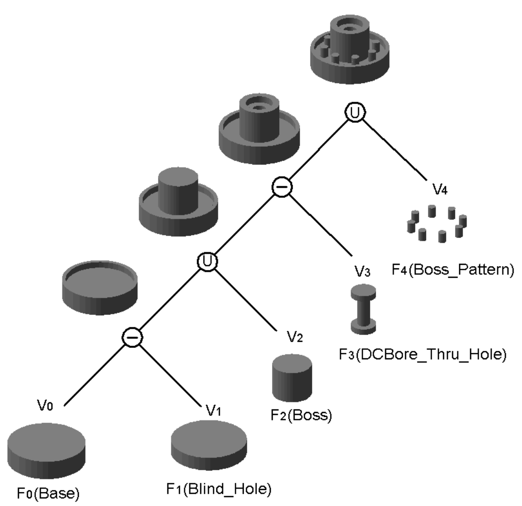


Fig. 1. An example of feature-based solid modelling.

### 2.1 A Merged Set in Non-Manifold Topology

A non-manifold cellular model is adopted as the topological framework for multi-resolution representation, and the merge & select algorithm is introduced for boundary evaluation. The non-manifold model can represent any combination of wireframe, surface, solid, and cellular models in a unified data structure, and Boolean operations are closed in the representation domain of non-manifold models, unlike the case for solid models [4, 12]. Several data structures have been proposed to represent non-manifold objects [9, 13, 19, 22, 24]. In this paper, the Partial Entity Structure [13] is adopted as the non-manifold topological framework. However, as all algorithms presented in this paper are written using only common topological entities, such as regions, faces, edges, and vertices, they can be implemented using any other non-manifold representations.

In the merge & select algorithm [7, 18], all primitives are merged into a single boundary representation, called a merged set. The merged set contains a complete description of the input primitives, all intersections between them, and historical information describing the origins of the entities with respect to the topological entities of the original primitives. In this system, all historical information is stored in the cell topological entities. A Boolean logic evaluator, whose input is the CSG representation and the history, selects merged set entities corresponding to the Boolean result. The user can modify Boolean operators or their order of occurrence easily by simply re-executing the selection process. The user can also select with a CSG tree that contains only a subset of the primitives in the merged set, and selectively filter out primitives from the final result without actually removing them from the merged set. Figure 2 shows a merged set composed of the five features from Figure 1. The properties of the merged set are very useful for feature-based multi-resolution modelling. Once the merged set of all features has been generated, any LOD models can be extracted very quickly from the merged set. Therefore, it is a challenging research issue to discover a proper set of Boolean operations acting on the feature volumes to define a LOD model.

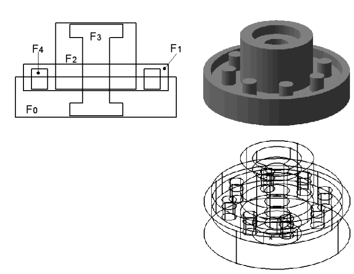


Fig. 2. A merged set of the features in Figure 1

## 2.2. Multi-Resolution Features

To make the feature-based multi-resolution modelling easier, we introduced a multi-resolution feature, whose attributes include all necessary information to build a multi-resolution model and extract LOD models. The multi-resolution features are arranged in the order of LOD. The attributes of the multi-resolution feature include the LOD, the pointer to the form feature, the creation order, the type of the Boolean operation, the name of the feature primitive, the effective volume of the feature, the error measure, and two cell topological entity lists $\Delta E_i^+$ and $\Delta E_i^-$, which store the differences between the *i*-th and (*i-1*)-th LOD models. Here, the cell topological entities represent the *0*-, *1*-, *2*-, and *3*-cells, which are equivalent to the vertex, edge, face, and region, respectively. The form feature for multi-resolution modelling will be discussed in more detail in Section 3. Here, $\Delta E_i^+$ and $\Delta E_i^-$ are investigated further, as they are used in the algorithm for building and extracting multi-resolution models.

Let $M$ denote the original solid model created by applying $n+1$ features $\{F_i\}_{i=0}^n$, and let $M_i \ (0 \le i \le n)$ denote the model at the *i*-th LOD. Then, $M_0$ is the lowest resolution model, and $M_n$ is the highest resolution model (this is equivalent to the original solid model, i.e. $M_n = M$). Let $E_i$ denote a collection of cell topological entities $e_j$ of $M_i$. Then $\Delta E_i^+$ and $\Delta E_i^-$ are defined respectively as:

$$\Delta E_i^+ = E_i - E_{i-1} \tag{1}$$

$$\Delta E_i^- = E_{i-1} - E_i \tag{2}$$

where $E_i = \left\{ e_j \mid e_j \in M_i, e_j \in \{0-,1-,2-, and\ 3-cells\} \right\}$

If $\Delta E_i^+$ and $\Delta E_i^-$ are given, $E_i$ can be obtained from $E_{i-1}$, and vice versa, using the following formulae:

$$E_i = E_{i-1} \cup \Delta E_i^+ - \Delta E_i^- \tag{3}$$

$$E_{i-1} = E_i - \Delta E_i^+ \cup \Delta E_i^- \tag{4}$$

The $\Delta E_i^+$ and $\Delta E_i^-$ parameters are stored in the multi-resolution feature, despite their storage cost, as they are useful for the fast extraction of LOD models.

When the example solid model in Figure 1 is created, a list of the multi-resolution features are filled as shown in the table in Figure 3. Here, the features are initially arranged in the order of feature creation and the effective volume of each feature is assigned to the name of its solid model. The whole volume of the feature is used by the Boolean operations to extract LOD models.

| No | LOD | Feature Name | Creation Order | Bool | Primitive | Effective Volume | Measure |
|---|---|---|---|---|---|---|---|
| 0 | 0 | Base | 0 | + | $V_0$ | $V_0$ | 0 |
| 1 | 1 | Blind_Hole | 1 | – | $V_1$ | $V_1$ | 0 |
| 2 | 2 | Boss | 2 | + | $V_2$ | $V_2$ | 0 |
| 3 | 3 | 2CBore_Thru_Hole | 3 | – | $V_3$ | $V_3$ | 0 |
| 4 | 4 | Boss_Pattern | 4 | + | $V_4$ | $V_4$ | 0 |

Fig. 3. Initial multi-resolution feature table for feature modelling in Figure 1

## 3. EFFECTIVE VOLUMES OF FEATURES

There are many ways in which features can be classified [21]. Feature taxonomies can be based on product categories, the intended applications of features, or feature shapes. Presently, there are no universally accepted or widely used feature taxonomies. Several taxonomy schemes have been proposed for classification entirely by shape; for example, Part 48 of STEP considers form features as consisting of three basic types: volume, transition, and pattern features [8]. A volume feature is an increment or decrement to the volume of a shape, such as a hole or a boss. A transition feature separates or blends surfaces, such as fillets or chamfers. A feature pattern is a set of similar features in a regular geometric arrangement, such as a circular or array pattern.

In this paper, form features are described using volumetric representations, and classified into additive and subtractive features. The three basic types of form features mentioned above are converted into volume features and reclassified into additive and subtractive features in the following manner. An incremental volume feature, such as a protrusion, connector, or stand-alone volume, is classified as an additive feature. A decremental volume feature, such as a passage, depression, or void, is classified as a subtractive feature. A transition feature is converted into a volume feature representing the volume subtracted or added to a part shape, and classified as a subtractive or additive feature accordingly. A feature pattern is also converted into a volume feature representing the resulting shape of the pattern. If the objective feature to which pattern is applied is additive, the volume feature is additive. Otherwise, the volume feature is subtractive.

The feature-based modelling process can be represented by a CSG tree, as illustrated in Figure 1. The terminal nodes of the tree describe the primitives of the features, while the internal nodes represent Boolean operations. The type of the feature determines the Boolean operation applied to it. If a feature is additive, the operation is union ($\cup$); if subtractive, the operation is difference ($-$).

For multi-resolution solid modelling, the features need to be rearranged according to a criterion that measures the significance of the feature. In general, more significant features appear at low LODs. However, if the features are rearranged, the resulting shape is possibly different from the original because union and subtraction Boolean operations are not commutative with each other. For example, if the features in Figure 1 are rearranged to $F_0 \to F_2 \to F_1 \to F_4 \to F_3$ , the result will be as shown in Figure 4, which is different from that expected. Therefore, it is crucial to find a method that, for an arbitrary rearrangement of features, results in the final shape being the same as the original shape, and with intermediate LODs of models having a reasonable shape.

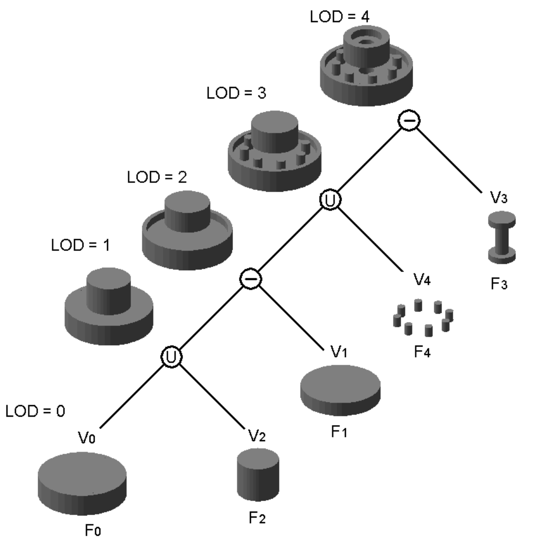


Fig. 4 A rearranged feature tree and its results

To achieve this goal, the concept of the effective volume of a feature is introduced. The region influenced by a Boolean operation is altered when the order of application of the operations is changed. For example, the union and difference Boolean operations do not follow the commutative law. Thus, to obtain the same resultant shape regardless of feature rearrangement, it is necessary to exclude some feature volumes from the original one. In this paper, this adapted volume is called the *effective volume* of a feature, or, alternatively, the *effective feature volume*. The following theorem illustrates how to identify this effective volume.

**Theorem 1 (Exchange of Two Features).**
If $V_i$ denotes the volume of the solid primitive of a feature $F_i$, and if $\otimes_i$ denotes the $\cup$ or – Boolean operation, then the result of exchanging two Boolean operations can be formularized as follows.

$$V_0 \otimes_1 V_1 \otimes_2 V_2 = V_0 \otimes_2 V_2 \otimes_1 \left(V_1 - \varphi(\otimes_1, \otimes_2)\, V_2\right) \qquad (5)$$

where

$$\varphi(a,b) = \begin{Bmatrix} 1 & \text{if } a \neq b \\ 0 & \text{if } a = b \end{Bmatrix} \qquad (6)$$

Here, the effective volumes of $F_0$, $F_1$, and $F_2$ are $V_0$, $V_1 - \varphi(\otimes_1, \otimes_2)\, V_2$, and $V_2$, respectively. To illustrate, an example model is shown in Figure 5. Assume that this model is created by applying three features in the sequence of $F_0 \to F_1 \to F_2$, and the order of features is then changed to $F_0 \to F_2 \to F_1$. If $F_0$ and $F_1$ are additive and $F_2$ is subtractive (i.e., $\otimes_1 = \cup$ and $\otimes_2 = -$ ), Equation (5) becomes $V_0 \cup V_1 - V_2 = V_0 - V_2 \cup (V_1 - V_2)$ . This case is illustrated in Figure 6, and the results demonstrate that this expression is true. In this case, the effective volumes of $F_0$, $F_1$, and $F_2$ are $V_0$, $V_1 - V_2$, and $V_2$, respectively. The reader can verify that Equation (5) is true for the other three cases: $\otimes_1 = \otimes_2 = \cup$ , $\otimes_1 = \otimes_2 = -$ , and $\otimes_1 = -$ and $\otimes_2 = \cup$ .

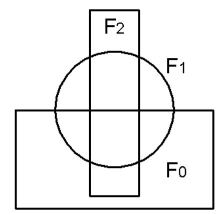


Fig.5 A model composed of three features

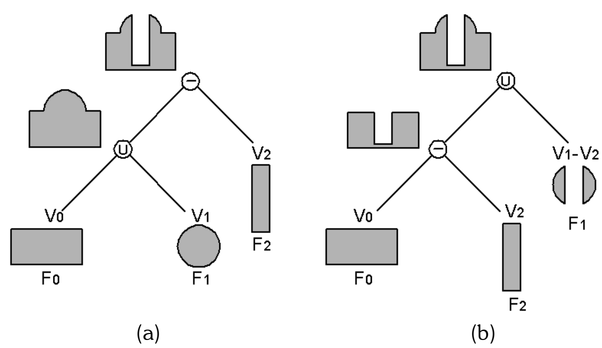


Fig.6 Effective feature volumes of the rearranged features: (a) the original feature tree, (b) a rearranged feature tree $F_0 \to F_2 \to F_1$, in which $V_0$, $V_1 - V_2$, and $V_2$ are the effective feature volumes of $F_0$, $F_1$, and $F_2$, respectively.

**Theorem 2 (Rearrangement of Features).**
Let $M_n$ denote the resulting model obtained by applying *n* Boolean operations between *n+1* solid models:

$$M_n = \prod_{i=0}^{n} \otimes_i V_i \text{ ,where } \otimes_0 V_0 = \varnothing \otimes_0 V_0 \quad (7)$$

If the *j*-th Boolean operation $\otimes_j V_j$ is moved to the *m*-th position, $M_n$ can be represented as follows.

$$M_n = \left( \prod_{i=0, i \neq j}^{m} \otimes_i V_i \right) \left( \otimes_j \left( V_j - \sum_{\ell=1}^{m-j} \varphi(\otimes_j, \otimes_{j+\ell}) V_{j+\ell} \right) \right) \left( \prod_{i=m+1}^{n} \otimes_i V_i \right)$$

where

$$\varphi(a,b) = \begin{Bmatrix} 0 \text{ if } a = b \\ 1 \text{ if } a \neq b \end{Bmatrix} \quad (8)$$

The proof of Theorem 2 is omitted in this paper. Instead, the example introduced in Figure 1 and Figure 2 is used to illustrate. As illustrated in Figure 7, moving the order of the feature $F_1$ to the last location results in the feature order $F_0 \to F_2 \to F_3 \to F_4 \to F_1$ and the Boolean sequence $V_0 \cup V_2 - V_3 \cup V_4 - (V_1 - V_2 - V_4)$ according to Equation (8). In this case, the effective volumes of $F_0$, $F_1$, $F_2$, $F_3$, and $F_4$ are $V_0$, $V_1 - V_2 - V_4$, $V_2$, $V_3$, and $V_4$, respectively.

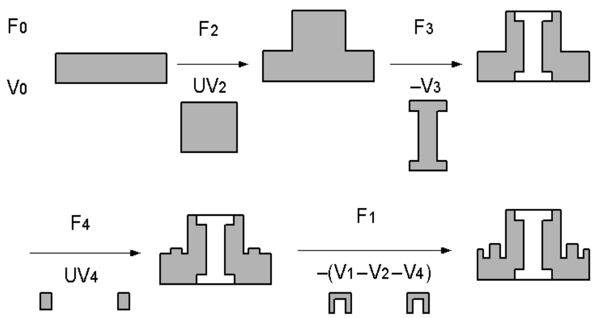


Fig.7 The effect of changing the order of features to $F_0 \to F_2 \to F_3 \to F_4 \to F_1$: the effective volumes of $F_0$, $F_1$, $F_2$, $F_3$, and $F_4$ are $V_0$, $V_1 - V_2 - V_4$, $V_2$, $V_3$, and $V_4$, respectively.

## 4. CONSTRUCTION AND EXTRACTION OF THE MULTI-RESOLUTION MODEL.

If the order of multi-resolution features is changed according to a given LOD criterion, the contents of multi-resolution features, such as the effective volumes, $\Delta E_i^+$, and $\Delta E_i^-$, are modified accordingly. The modified multi-resolution features allow extraction of the solid boundary representation for a given LOD. The method of updating the data of multi-resolution features for a new feature order is introduced in Algorithm 1.

In this algorithm, the most detailed feature is first selected and moved to the *n*-th place. Next, the secondly most detailed feature is selected and moved to the *n-1*-th place. This is repeated until the most significant feature is located at the *0*-th place. Whenever each feature is moved to its new place, its new effective volume is redefined according to Equation (8). In the implementation of this algorithm, the definition of an effective volume is actually stored as a string of characters.

Once the feature rearrangement is complete, $\Delta E_i^+$ and (8) $\Delta E_i^-$ are updated for each feature. For the *i*-th multi-resolution feature, a sequence of Boolean operations on the *0*-th to *i*-th effective volumes are described as a character string, which is used to construct a CSG tree. The cell topological entities constituting the corresponding LOD model are searched next, and marked in the merge & select algorithm. Finally, the differences between the current and the previous LOD model, $\Delta E_i^+$ and $\Delta E_i^-$, are saved in the multi-resolution feature.

**Algorithm 1. UpdateMultiresolutionFeatures** (*F*, *mset*)

```
Input: F: the multiresolution feature list. F = {F_i}_{i=0}^n .
       mset: the merged set model of the features.
Output: F: the reordered list of the multiresolution features.
for k ← n to 1 do {
   // Find out the most detailed feature F_j from {F_i}_{i=0}^k .
   j←FindLeastSignificantFeature (F, 0, k).
   // Move the feature F_j to k-th position.
   MoveMultiresolutionFeature(F, j, k).
}
// Select and store the cell entities e_j of M_n in the entity list E_n.
E_n ← { e_j | e_j ∈ M_n, e_j ∈ M, M_n = ∏_{i=0}^{n} ⊗_i V_i }
for k ← n to 1 do {
   // Select and store the cell entities of M_{k-1} in the list E_{k-1}.
   E_{k-1} ← { e_j | e_j ∈ M_{k-1}, e_j ∈ M, M_{k-1} = ∏_{i=0}^{k-1} ⊗_i V_i }
   // Set the differences between E_k and E_{k-1}.
   ΔE_k^+ ← E_k − E_{k-1} .
   ΔE_k^- ← E_k − E_{k-1} .
}
return F.
```

**Algorithm 2. MoveMultiresolutionFeature**(*F, j, m*)

```
Input:   F: a multiresolution feature list, F = {F_i}_{i=0}^n .
         j: the current location of the feature to be moved.
         m: the destination location of the feature F_j.
Output:  F: the rearranged feature list.
Set a character string S with the effective volume V_j of F_j.
for k ← 1 to m-j do {
   if the type ⊗_{j+k} of F_{j+k}, is not equal to ⊗_j of F_j then {
      Get the effective volume V_{j+k} of F_{j+k} in a character string.
      S ← S+"−"+V_{j+k} .
   }
}
// Set the effective volume V_j of F_j with S.
V_j ← S .
// Move  F_j to the m-th place
F_tmp ← F_j ; for k ← j+1 to m do { F_{k−1} ← F_k };
F_m ← F_tmp
```

Once a multi-resolution model containing a merged set and a reordered multi-resolution feature list is generated, it is possible to extract LOD models. If the current LOD is $i$, and the desired LOD is $k$, then a collection of the cell topological entities $E_k$ of the $k$-th LOD model $M_k$ can be obtained from $E_i$ using the following formulas, which are derived from Equations (3) and (4).

$$if\, k > i, E_k = E_i \cup \sum_{j=i+1}^{k} \Delta E_j^{+} - \sum_{j=i+1}^{k} \Delta E_j^{-} \tag{9}$$

$$if\, k < i, E_k = E_i - \sum_{j=i-1}^{k} \Delta E_j^{+} \cup \sum_{j=i-1}^{k} \Delta E_j^{-} \tag{10}$$

## 5. CRITERIA OF LEVEL OF DETAIL

### 5-1. VOLUMES OF SUBTRACTIVE FEATURES

The criteria of LOD are application-dependent; the volume of the feature is one possible LOD criterion. In past research, additive features have taken precedence over all subtractive features, and subtractive features were then rearranged in the descending order of their volumes [5, 11]. The lowest resolution model is a volume enclosing all additive features, and the higher resolution models are obtained by removing the volumes of subtractive features from the enclosing volume in descending order. This method has been used for applications such as rendering and streaming solid models. It is noted that the proposed multi-resolution modelling approach can support this LOD criterion.

Let it be assumed that the solid model of a part is created by applying $n+1$ features: $k+1$ additive features and $n$-$k$ subtractive features. First, the features are separated into additive and subtractive groups, with the additive feature group taking precedence over the subtractive feature group. Next, the features in each group are rearranged in the descending order of the feature volumes. The rearrangement of the features can be represented by Equation (11), where $\otimes_i'$ and $V_i'$ are the Boolean operator and the effective volume of the $i$-th feature after the feature rearrangement, respectively.

$$M_n = \prod_{i=0}^{n} \otimes_i' V_i' = \left( \prod_{i=0}^{k} \otimes_i' V_i' \right) \left( \prod_{i=k+1}^{n} \otimes_i' V_i' \right), 0 \le k \le n \tag{11}$$

As the lowest resolution model $M_0$ is the union of all of the $k+1$ additive features, the number of LODs is reduced to $n$-$k+1$. A series of LOD models $M_0, M_1, \ldots, M_{n-k}$ can be obtained as follows.

$$M_0 = \prod_{i=0}^{k} \left( \cup V_i' \right) \tag{12}$$

$$M_j = M_0 \prod_{i=k+1}^{k+j} \left( -V_i' \right), \; 1 \le j \le n-k \tag{13}$$

If this criterion is applied to the example in Figure 1, the features are rearranged as $F_0 \to F_2 \to F_4 \to F_1 \to F_3$ . The resulting multi-resolution features are illustrated in Figure 8. Because three features $F_0$, $F_2$, and $F_4$ are of the additive type ($k=2$), the lowest LOD model is their union set. As a result, three different LOD models shown in Figure 9 can be extracted from this multi-resolution model: $M_0 = V_0 \cup V_2 \cup V_4$ , $M_1 = M_0 - (V_1 - V_2 - V_4)$ , and $M_2 = M_1 - (V_3 - V_4)$ .

| No | LOD | Feature Name | Creation Order | Bool | Primitive | Effective Volume | Measure |
|---|---|---|---|---|---|---|---|
| 0 | * | Base | 0 | + | $V_0$ | $V_0$ | 42412 |
| 1 | * | Boss | 2 | + | $V_2$ | $V_2$ | 17671 |
| 2 | 0 | Boss_Pattern | 4 | + | $V_4$ | $V_4$ | 2036 |
| 3 | 1 | Blind_Hole | 1 | − | $V_1$ | $V_1 - V_2 - V_4$ | 22902 |
| 4 | 2 | 2CBore_Thru_Hole | 3 | − | $V_3$ | $V_3 - V_4$ | 1963 |

Fig. 8 Reordered multi-resolution feature table for the example solid model in Figure 1: the level of detail (LOD) criterion is the volume of the feature, together with the precedence of additive features over subtractive features.

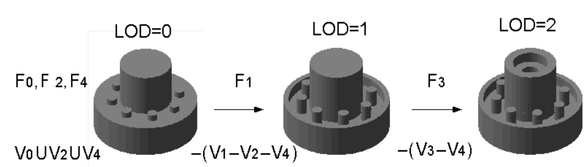


Fig. 9 Three LOD models for the reordered multi-resolution features in Figure 8.

The LOD criterion used in this method is the whole volume of the subtractive feature. However, the whole volume of the subtractive feature may not used to subtract the feature from the part model. Only the volume $V_i \cap (M_0 - M_n)$ contributes to the Boolean result, thus this intersection volume may be a more reasonable LOD criterion than the whole feature volume. Moreover, this method has a few more serious problems as a general LOD criterion. First, some additive features can be more detailed than subtractive features, meaning that the lowest resolution model may have a more detailed shape than higher resolution models. Second, if a part is modelled using only additive features, only one LOD is possible. To overcome these problems, in this paper the volume of feature is proposed as a new criterion. This criterion is more general, and works regardless of whether the feature is additive or subtractive.

## 5-2. Volumes of Features

The criterion of the volume of feature can be refined, according to which volume is being measured. The simplest case is to use the volume of the feature itself. However, as mentioned above, this may include volume that does not contribute to the final shape. The method presented in Algorithm 3 (which is called by Algorithm 1) is proposed to solve this problem. Algorithm 3 calculates the volume of the model generated by omitting features in turn from a given list of features $\{F_i\}_{i=\ell}^{k}$. Let $V_m$ denote the volume of the model that is generated by applying all features $\{F_i\}_{i=\ell}^{k}$ except the feature $F_m$. If $V_m$ is the closest to the reference volume $V_{ref}$, then the feature $F_m$ is the most detailed feature among $\{F_i\}_{i=\ell}^{k}$. The reference volume can be that of the original part model $M(=M_n)$ or the $k$-th LOD model $M_k$. Here, the volume of the original part model $M_n$ is selected as the reference volume.

**Algorithm 3**. **FindLeastSignificantFeature** ($F, \ell, k$)
**Input**: $F$: the multiresolution feature list: $F = \{F_i\}_{i=0}^{n}$.
$\ell, k$: lower and upper bounds of the feature range for searching the least significant feature: $\{F_i\}_{i=\ell}^{k}, \ell \le i \le k, 0 \le \ell, k \le n$.
**Output**: returns the position of feature of minimum volume.
// Set the variable $min\Delta V$ to a huge value.
$min\Delta V \leftarrow \infty$.
// Set the reference volume size $V_{ref}$.
$V_{ref} \leftarrow$ **VolumeSize**($M_n$).
**for** $i \leftarrow \ell$ **to** $k$ **do** {
$\Delta V = \| V_{ref} - \mathbf{VolumeSize}\left(\prod_{j=\ell, j\ne i}^{k} \otimes_j V_j\right) \|$.
**if** ($\Delta V < \min\Delta V$) **then** {
$min\Delta V \leftarrow \Delta V$.
$min\_position \leftarrow i$.
}
}
return *min_position*.

If this algorithm is applied to the example in Figure 1, the features are rearranged in the order $F_0 \to F_2 \to F_1 \to F_4 \to F_3$. Figure 10 shows the resulting multi-resolution feature table. The effective volumes of $F_0$, $F_1$, $F_2$, $F_3$, and $F_4$ are $V_0$, $V_1 - V_2$, $V_2$, $V_3 - V_4$, and $V_4$, respectively. As illustrated in Figure 11, five LOD models can be extracted from this multi-resolution model.

| No | LOD | Feature Name | Creation Order | Bool | Primitive | Effective Volume | Measure |
|---|---|---|---|---|---|---|---|
| 0 | 0 | Base | 0 | + | $V_0$ | $V_0$ | * |
| 1 | 1 | Boss | 2 | + | $V_2$ | $V_2$ | * |
| 2 | 2 | Blind_Hole | 1 | − | $V_1$ | $V_1 - V_2$ | * |
| 3 | 3 | Boss_Pattern | 4 | + | $V_4$ | $V_4$ | * |
| 4 | 4 | 2CBore_Thru_Hole | 3 | − | $V_3$ | $V_3 - V_4$ | * |

Fig.10 Reordered multi-resolution features for the example solid model in Figure 1, using feature volume as the LOD criterion.

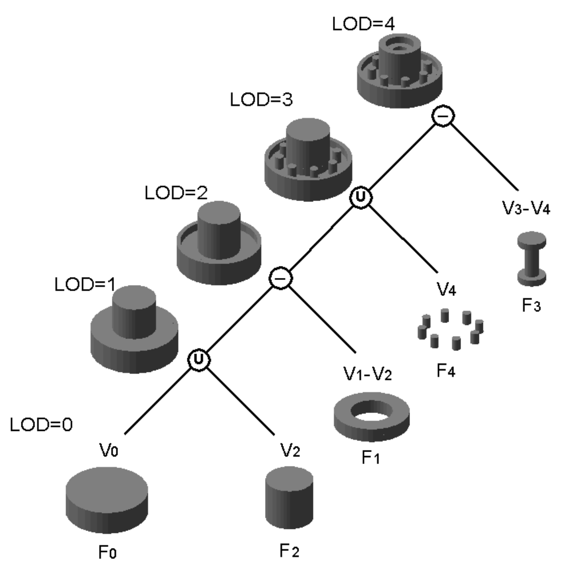


Fig.11 The LOD models according to the multi-resolution feature table in Figure 10.

## 6. CONCLUSIONS

In this paper, a new approach for multi-resolution modelling for feature-based solid models is proposed. The characteristics and contributions of this new approach are summarized as follows.

- In most previous work, the triangular mesh is the object of multi-resolution modelling, and applications are mainly focused on fast rendering and transmitting geometric models in computer graphics. Low-level topological entities, such as vertices, edges, or faces, are removed or suppressed to generate low-resolution models. However, in this work, the solid model is used as the object of multi-resolution modelling. Applications are mainly focused on engineering tasks, such as analysis, network-based collaborative design, and virtual prototyping and manufacturing. The suppression objects are form features that are a higher level of modelling entities than the topological entities.
- In multi-resolution solid modelling, one challenge is to generate valid LOD models after an arbitrary rearrangement of features according to a certain LOD criterion. In general, if features are rearranged, the resulting shape is different from the original one, because the union and subtraction Boolean operations are not commutative to each other. This paper proposed the concept of using the effective feature volume, and developed an algorithm for feature-based multi-resolution modelling based on the effective volume. The multi-resolution feature was also newly defined to facilitate the implementation of the algorithm. This algorithm guarantees the same resulting shape and the reasonable intermediate LOD models for an arbitrary rearrangement of the features, regardless of whether feature types are additive or subtractive. This characteristic enables various LOD criteria to be used for a wide range of applications.
- The non-manifold merged set model was adopted for the multi-resolution representation of feature-based solid models, and the merge & select algorithm was introduced for boundary evaluation. Since the merged set contains all boundary information about features, various LOD models can be extracted in a short time. Moreover, by storing the boundary information for LOD models in the multi-resolution features, transition to any LOD can be performed immediately without the select process.
- The effective volume of the feature is independent of the data structure of the model. Although the multi-resolution modelling algorithm was implemented based on the non-manifold representation, it can also be implemented using the conventional B-rep solid representation.
- The volume of the feature, regardless of feature type, was proposed as a new criterion of LOD. The contribution of a feature to the final part shape is measured more precisely by examining the amount of volume removal or addition to the part for each feature modelling. This criterion can be used in a wide range of applications, since there is no distinction between additive and subtractive features unlike the previous method.

The following are proposed as future research topics:

- *To extend the multi-resolution modelling technique to engineering analysis*: Geometric models for analysis are simplified and idealized models that may include not only solid models but also medial surfaces and wireframes. Depending on an engineer's intent and the desired accuracy of analysis results, geometric models at various levels of abstraction need to be provided for CAE systems. Extension of the representation domain from solid to non-manifold models is required to support the multi-resolution analysis model in the mixed dimension.
- *To generate multi-resolution representations for assembly models*: This is useful for virtual prototyping and manufacturing in particular. Assembly constraints should be considered when generating multi-resolution assembly models.
- *To propose new LOD criteria for new applications*: The LOD criteria proposed so far are the volume of the subtractive feature and the volume of the feature. More LOD criteria may need to be suggested for more applications, as LOD criteria are usually application-dependent.


## 7. ACKNOWLEDGEMENTS

This work was supported by grant No. R01-2002-000-00061-0 from the Basic Research Program of the Korea Science & Engineering Foundation.

## 8. REFERENCES


[1] Armstrong, C. G., Modelling requirements for finite-element analysis, *Computer-Aided Design* Vol. 26, No. 7, 1994, pp. 573-578.

[2] Belaziz, M., Bouras, A. and Brun, J. M. Morphological analysis for product design, *Computer-Aided Design*, Vol. 21, No. 5-6, 2000, pp. 377-388.

[3] Bidarra, R., Kranendonk, N., Noort, A. and Bronsvoort W. F., A collaborative framework for integrated part and assembly modeling, in *Proceedings of the Seventh ACM Symposium on Solid Modeling and Applications*, Saarbrücken, Germany, June 17-21, 2002: pp. 389-400.

[4] Charlesworth, W. W. and Anderson, D. C., Applications of non-manifold topology, in *Proceedings of International Computers in Engineering Conference and the ASME Database Symposium*, Boston, Massachusetts, September 17-20, 1995, pp. 103-112.

[5] Choi, D. H., Kim, T. W. and Lee, K., Multiresolutional representation of b-rep model using feature conversion, *Transactions of the Society of CAD/CAM Engineers*, Vol. 7, No. 2, 2002, pp.121-130. (in Korean)

[6] Cignoni, P., Montani, C. and Scopigno, R., A comparison of mesh simplification algorithms, *Computers & Graphics,* Vol. 22, No. 1, 1998, pp. 37-54.

[7] Crocker, G. A. and Reinke, W. F. An editable non-manifold boundary representation, *IEEE Computer Graphics & Applications,* Vol. 11, No. 2, 1991, pp. 39-51.

[8] Dunn, M., Industrial automation systems and integration - product data representation and exchange – part 48: integration generic resources: form features, Second edition. ISO/WD 10303-48.

[9] Gursoz, E. L., Choi, Y. and Prinz F. B., Vertex-based boundary representation of non-manifold boundaries. In: Wozny, M.J., Turner, J.U. and Preiss K., editors, *Geometric modeling for product engineering*, North-Holland, 1990, pp.107-130.

[10] Koo, S. and Lee, K., Wrap-around operation to make multi-resolution model of part and assembly, *Computers & Graphics*, Vol. 26, No. 5, 2002, pp. 687-700.

[11] Lee, J. Y., Lee, J.-H., Kim, H. and Kim, H. S., A cellular topology-based approach to generating progressive solid models from feature-centric models, *Computer-Aided Design*, Vol. 36, No. 3, 2004, pp. 217-29.

[12] Lee, K., *Principles of CAD/CAM/CAE Systems*, Addison Wesley Longman, Inc., 1999.

[13] Lee, S. H. and Lee, K., Partial entity structure: a compact boundary representation for non-manifold geometric modeling, *ASME Journal of Computing & Information Science in Engineering*, Vol. 1, No. 4, 2001, pp. 356-365.

[14] Lee, S. H., Lee, K.-S. and Park, S., Multiresolution representation of solid models using the selective boolean operations, in *Proceedings of 2002 Spring Conference of Korean Society of Precision Engineering*, Daejeon, Korea, May 17-18, 2002, KSPE 02S179, pp. 833~835. (in Korean)

[15] Li, B. and Liu, J., Detail feature recognition and decomposition in solid model, *Computer-Aided Design,* Vol. 34, No. 5, 2002, pp. 405-414.

[16] Li, J., Gao, S. and Zhou, X., Direct incremental transmission of boundary representation. in *Proceedings of the 8th ACM Symposium on Solid Modeling and Applications*, Seattle, Washington, USA, June 16-20, 2003, pp. 298-303.

[17] Lin, S.-Y. T., Lin, Y.-F., *Set theory: an intuitive approach*, Boston, Houghton Mifflin, 1974.

[18] Masuda, H., Topological operators and boolean operations for complex-based nonmanifold geometric models, *Computer-Aided Design,* Vol. 25, No. 2, 1992, pp. 119-129.

[19] Rossignac, J. And O'Conner, M. A., SGC: a dimensional-independent model for pointsets with internal structures and incomplete boundaries, In: Wozny, M.J., Turner, J.U. and Preiss, K., editors, *Geometric modeling for product engineering*, North-Holland, 1990, pp. 145–180.

[20] Schröder, W. J., Zerge, J. A. and Lorensen, W. E., Decimation of triangle meshes, in *Proceedings of SIGGRAPH '92* in *Computer Graphics*,Vol. 26, No. 2, 1992, pp. 65-70.

[21] Shah, J. J. and Mäntylä, M., Parametric and feature-based CAD/CAM, John Wiley & Sons, Inc., 1995.

[22] Weiler, K., The radial edge structure: a topological representation for non-manifold geometric boundary modeling. In: Wozny M. J., et al, editors, *Geometric Modeling for CAD Applications*, Noth-Holland, 1988,pp.3-36.

[23] Wu, D., Dhargava, S. and Sarma, R., Solid model streaming as the basis for a distributed design environment, in *Proceedings of the 2000 ASME Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, 26th Design Automation Conference, Baltimore, Maryland, September 10-13, 2000, DETC2000/DAC-14250.

[24] Yamaguchi, Y. and Kimura, F., Nonmanifold topology based on coupling entities, *IEEE Computer Graphics and Applications*, Vol. 15, No. 1, 1995, pp. 42-50.