History-Based Selective Boolean Operations for Feature-Based Multi-resolution Modeling

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Abstract. The feature-based multi-resolution models of a solid represent an object at different levels of detail (LOD), particularly in the unit of feature, according to a certain LOD criterion. The need of feature-based multi-resolution modeling is currently increasing for various engineering tasks, such as analysis, network-based collaborative design, virtual prototyping and manufacturing. To provide feature-based multi-resolution models, it is essential to generate a valid and unique solid model at each LOD after feature rearrangement. To meet this requirement, we propose the *history-based selective Boolean operations* that satisfy the commutative law between union and subtraction by considering the history of the Boolean operations for the features. Owing to the commutative properties, the feature-based multi-resolution modeling technique based on these operations guarantees the same resulting shape as the original, and provides a unique and reasonable shape at each intermediate LOD after arbitrary feature rearrangement.

1 Introduction

Three-dimensional (3D) computer aided design (CAD) systems based on feature-based solid modeling techniques are widespread and commonly used for product design. However, when part models associated with features are used in various downstream applications, simplified models with various level of detail (LOD) are frequently more desirable than the full details of the parts. In particular, there is increasing need in engineering tasks for feature-based multi-resolution representations of a solid model, which represents an object using multiple LODs in the unit of feature, such as in analysis [1, 6], network-based collaborative design [5], and virtual prototyping and manufacturing [4].

To respond to this need, several researchers have examined feature-based multi-resolution modeling techniques. Choi et al. [2] studied multi-resolution representation of a B-rep part model. In their approach, the lowest resolution model was formed by uniting all of the additive features, and then higher resolution models were generated by applying the subtractive features successively in descending order of volume. This

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method has the advantage in that it can be implemented in current commercial 3D CAD systems by virtue of sharing the same data structure. However, this method has several limitations. First, it requires a large volume of computation time to obtain an LOD model, as Choi et al. used conventional Boolean operations. (Here, an *LOD model* denotes a solid model at a specific LOD). Second, the method of Choi et al. cannot provide adequate LOD models for an arbitrary feature rearrangement, regardless of the feature type, which is additive or subtractive.

To reduce the computation time for the extraction of LOD models, Lee [7] and Lee, et al. [5] introduced the nonmanifold topological model of a cellular structure as the topological framework for a multi-resolution model. In this method, all the features are initially merged into an non-manifold cellular model, and then, if the LOD is given, the topological entities comprising the LOD model are selected and displayed. In particular, Lee et al. [5] addressed the incremental transmission of solid models for engineering tasks through a network in order to share the model at an adequate LOD. Kim, et al. [3] introduced a feature extraction method and proposed three operators to build multi-resolution models. Three operators include wrap-around, smooth-out, and thinning operators. However, it is not clear their proposed set of multi-resolution features and extraction operators are complete to perform multi-resolution modeling.

Recently, Lee [7] introduced the concept of an *effective volume of a feature* to provide valid solids for an arbitrary rearrangement of features, regardless of the feature type. The effective volume of a feature was defined as the actual volume of the feature in the rearranged feature tree when used as a tool body for the Boolean operation. When arranged in order of feature creation, the effective volume of each feature was defined as the entire volume of the feature. However, the effective volume of a feature can be reduced to a fraction of the original volume after feature rearrangement. Lee described a method to identify the effective volume and provided a mathematical proof of the method's correctness. By introducing the concept of an effective volume, an arbitrary rearrangement of features becomes possible, and arbitrary LOD criteria may be selected to suit various applications. However, the effective volume may be defined differently according to the order of relocation of the features. If the order is not selected properly, then some intermediate LOD models may have unacceptable shapes. (This problem is addressed in Section 2 in this paper in more detail). Therefore, a method to guarantee a reasonable and unique shape for each intermediate LOD needs to be developed.

To solve the feature rearrangement problem completely, we propose using history-based selective Boolean (HBS-Boolean) operations, and applying them to feature-based multi-resolution modeling. Since union and subtraction are commutative in HBS-Boolean operations, our approach guarantees the same result for an arbitrary rearrangement of features, and unique and reasonable shapes for each LOD. Since the HBS-Boolean operations are independent from the topological framework, they can be implemented using conventional solid data structures as well as nonmanifold data structures.

The remainder of the paper is organized as follows. Section 2 defines the problem. Section 3 introduces a definition of the union and subtraction operations of the HBS-Boolean operations, and describes their properties. Section 4 describes the implementation of HBS-Boolean operations, and discusses a case study, and some conclusions and future work are given in Section 5.

2 Problem Definition

Let F, P, and $\otimes$ denote a feature, the primitive of a feature, and the $\cup$ or $-$ Boolean operation of a feature, respectively. If X is one of F, P, or $\otimes$, then X^i denotes the i-th X in the initial feature creation order, while X_j denotes the j-th X in the current rearranged order. Since P is defined as a point set over the R^3 space in this work, it includes the solid and the 3D nonmanifold topological model.

If M_n denotes the resulting model of $n+1$ features, then it is created by applying n Boolean operations between the $n+1$ primitives of the features, $M_n = \prod_{k=0}^{n} \otimes^k P^k$, where $\otimes^0 P^0 = \phi \otimes^0 P^0$. However, if the features are rearranged, then the resulting shape, denoted by M'_n ($= \prod_{\ell=0}^{n} \otimes_\ell P_\ell$), is generally different from the original shape because union and subtraction Boolean operations are not commutative with each other. To apply feature-based multi-resolution modeling to a wide range of application areas, even though the features are rearranged arbitrarily regardless of whether the feature is additive or subtractive, the resulting shape must be the same as the original shape, and the models at the intermediate LODs must have a reasonable shape. This problem is denoted as the feature rearrangement problem.

To solve this problem, we introduced the concept of an effective volume of a feature [7]. The region influenced by a Boolean operation is altered when the order of application of the operations is changed. Thus, to obtain the same resultant shape regardless of feature rearrangement, it is necessary to exclude some feature volumes from the original shape. This adapted volume is called the effective volume of a feature, or, alternatively, the effective feature volume. The effective volumes of all the features in M_n are described by Eq. (1) when a feature, F_i, is relocated to the m-th place.

$$M_n = \left(\prod_{k=0,k\neq i}^{j} \otimes_k P_k \right) \otimes_i \left(P_i - \sum_{\ell=0}^{j-i} \psi(\otimes_i, \otimes_{i+\ell}) P_{i+\ell} \right) \left(\prod_{k=j+1}^{n} \otimes_k P_k \right), \ i < j \tag{1}$$

where

$$\psi(a,b) = \begin{Bmatrix} 1 & if \ a \neq b \\ 0 & \text{otherwise} \end{Bmatrix}.$$

For an arbitrary feature rearrangement, the feature relocation operation using Eq. (1) guarantees the same resulting shape as the original. However, the intermediate LOD models are altered, depending on the order of the feature relocation operations. Moreover, there is no criterion to decide which is the most reasonable shape among the various options.

Let us investigate this problem using the example model shown in Fig. 1. This model has been created by applying four features, as illustrated in Fig. 1.

Let us rearrange the features in the order: $F^0 \to F^2 \to F^1 \to F^3$, and calculate the effective volumes of the rearranged features using Eq. (1). If the features are moved in the manner shown in Figs. 2(a) and 3(a), then the resulting LOD models are shown in Figs. 2(b) and 3(b), respectively. Although the resulting shapes of the two cases are similar, the intermediate LOD models of the two cases are different. The LOD models in Fig. 2 appear unreasonable, while those in Fig. 3 appear reasonable. In the case of Fig. 2, the excessive part of the feature effective volume is removed, as on applying F^2 and F^1, part of the volume, P^3, is subtracted unnecessarily in advance.

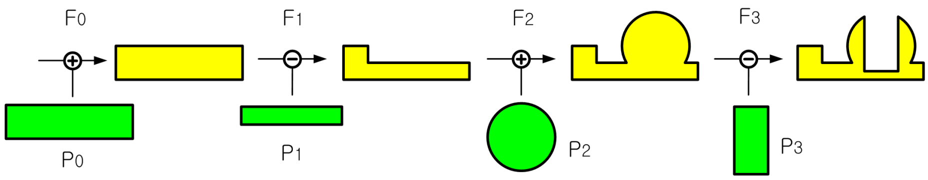

Fig. 1. An example of feature-based solid modeling: (a) the part model and its form features, and (b) the feature modeling tree

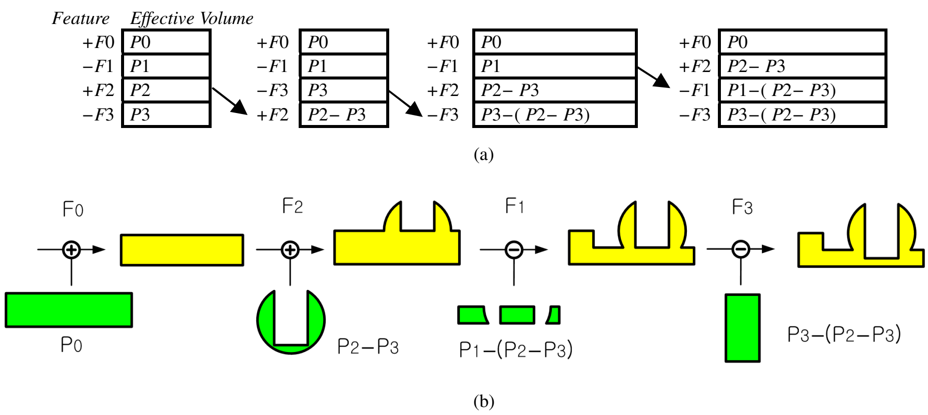

Fig. 2. A feature relocation process and the result for the feature rearrangement: $F^0 \rightarrow F^2 \rightarrow F^1 \rightarrow F^3$. (a) The series of feature relocations, and (b) the LOD models resulting from the feature relocation process in shown in 2(a).

As shown in Figs. 2 and 3, the intermediate LOD models may alter, depending on the order of the feature relocation operations. The reason for this, is that at each operation, the effective volume of the moved feature is calculated using the current definition of the effective volume. Therefore, to form intermediate LOD models with an acceptable shape using the operation based on Eq. (1), this order must be selected carefully. Moreover, there is no criterion that can be used to decide which is the most reasonable shape for an LOD model among the various options. Therefore, it is crucial to devise a method that guarantees a reasonable and unique shape for each intermediate LOD model independent of the order of the feature relocation operations.

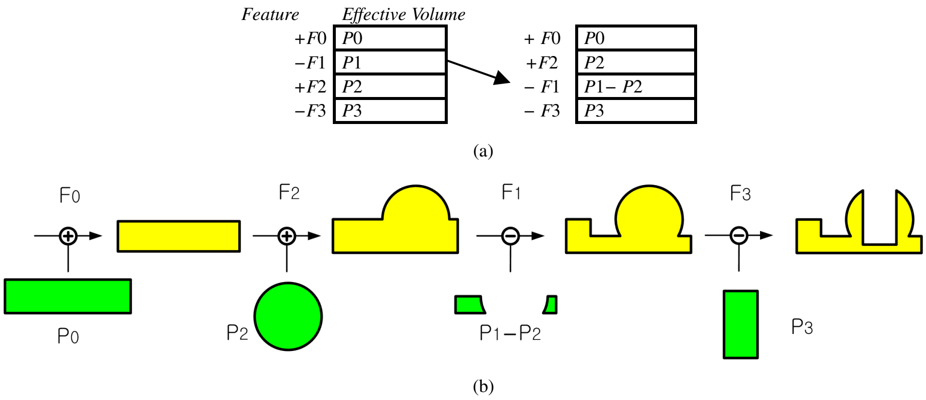

Fig. 3. Another feature relocation process and the result for the feature rearrangement: $F^0 \rightarrow F^2 \rightarrow F^1 \rightarrow F^3$. (a) The series of feature relocations, and (b) the LOD models resulting from the feature relocation process in shown in 3(a).

3 History-Based Selective Boolean Operations

To meet this requirement, we proposed *history-based Boolean* (*HBS-Boolean*) *operations* that obey commutative laws for union, subtraction, and intersection operations. By virtue of the commutative property of the union and subtraction operations, for an arbitrary feature rearrangement, these operations guarantee the same resulting shape as the original, and also a unique and acceptable shape at each intermediate LOD independent of the order of the feature movements. Although HBS-Boolean operations include intersection, we defined and will discuss only union and subtraction in this work because in general, feature-based modeling is implemented using only union and subtraction operations.

3.1 Definition

When the order of the Boolean operations is changed, the resulting shape may be different from the original. This phenomenon is due to the following reasons: (1) mixed union and subtraction operations do not obey the commutative law, and (2) Boolean operations are always applied to the entire shape, and thus, the region affected by each Boolean operation in the initial creation order is different from the affected region in the rearranged order.

In the HBS-Boolean operation, the volume of each feature is refined considering the above reasons to provide the same resulting shape, as well as a reasonable and unique shape at each intermediate LOD. The refinement is conducted by excluding the overlapping volume of the feature that satisfy the following conditions from the volume of each feature:

– To be located at the post-position in the initial creation order, but at the pre-position in the rearranged order, and
– To be of a different feature type, which is additive or subtractive.

The definition of the HBS-Boolean operations can be formalized from the refinement method above. Let F_j^i denote a feature that is applied at the i-th place in the original order, but is now located at the j-th place in the rearranged order. If $\hat{\otimes}$ denotes an HBS-Boolean operation, then the corresponding HBS-Boolean operation of F_j^i is $\hat{\otimes}_j^i P^i$. If Z_j^i denotes the refined volume of F_j^i, then the HBS-Boolean operation $\hat{\otimes}_j^i P^i$ can be represented by Eq. (2)

HBS-Boolean Operation: $\hat{\otimes}_j^i P^i = \otimes^i Z_j^i$, (2)

where

$$Z_j^i = P^i - \sum_{\ell=0}^{j-1} \varphi(j,\ell)\gamma(i,k(\ell))P_\ell^{k(\ell)}, \tag{3}$$

and where $\varphi(i,j) = \begin{Bmatrix} 1 & if\ \otimes_i \neq \otimes_j \\ 0 & \text{otherwise} \end{Bmatrix}$, $\gamma(i,j) = \begin{Bmatrix} 1 & if\ i < j \\ 0 & \text{otherwise} \end{Bmatrix}$, and $k(\ell)$ is the initial location (in the original order) of the current ℓ-th feature (in the rearranged order).

3.2 Properties

The HBS-Boolean operations have the following properties:

- The resulting shape of the rearranged HBS-Boolean operations is the same as the original.
- The shape of each intermediate LOD model is determined uniquely and independently of the order of the feature relocation operations.
- Different types of HBS-Boolean operations obey the commutative law.
- The result of the HBS-Boolean operations is the same as that of the feature rearrangement algorithm that relocates the features in turn from the least significant feature in proportion to the feature significance.

Since the first four properties can be derived by the final property, we will investigate the final property first, and then prove the other properties using it.

3.2.1 Interpretation Using the Feature Rearrangement Algorithm

According to the feature rearrangement method of Algorithm 1 in Ref. [7], the least significant feature is selected first, and moved to the *n*-th place. Next, the second least significant feature is selected, and moved to the *n-1*-th place. This is repeated until the most significant feature is located at the *0*-th place. Whenever each feature is moved to its new place, the new effective volume is redefined using Eq. (1). Let us follow this algorithm.

Let $\overset{(k)}{P}_j$ denotes the feature primitive that is located at the j-th position in the k-th step. In the m-th step, let us assume that the m-th selected feature is located at an arbitrary y -th place, $(0 \le y \le n-m+1)$, in the step- $(m-1)$ order, and its primitive is P^i . If the feature is moved to the $n-m+1$ -th place, then its effective volume becomes

$$Z^i_{n-m+1} = \overset{(m-1)}{P^i_y} - \sum_{\ell=y+1}^{n-m+1} \overset{(m-1)}{\varphi} (y,\ell) \overset{(m-1)}{P_\ell} = \overset{(m)}{P^i_{n-m+1}} - \sum_{\ell=y}^{n-m} \overset{(m)}{\varphi} (y,\ell) \overset{(m)}{P_\ell}$$

$$= P^i_{n-m+1} - \sum_{\ell=0}^{n-m} \varphi(n-m+1,\ell)\gamma(i,k(\ell))P_\ell^{k(\ell)} \tag{4}$$

When this process is repeated from Step *1* to Step n , all the effective volumes of the rearranged features are eventually determined. It should be noted that in Step m , P^i is moved to the j -th place in the step- $(m-1)$ order. Then j becomes. If $m = n - j + 1$ and $P^i_j = P^i$ are applied to Eq. (4), then the result is the same $j = n - m + 1$ as obtained using Eq. (3). This means that the result of the HBS-Boolean operations is always the same as that of Algorithm 1.

Figure 4 shows the result of the HBS-Boolean operations when the features are moved following the feature relocation sequence of Fig. 2(a). This example shows that HBS-Boolean operations guarantee the same resulting shape as the original, and provide a reasonable shape at the intermediate LODs independent of the order of the feature relocation operations.

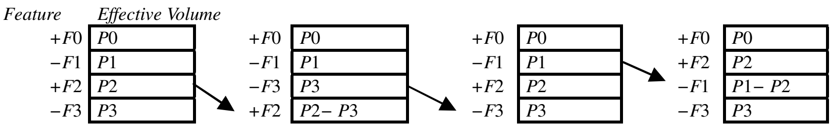

Fig. 4. The feature relocation process and the result using the HBS-Boolean operations for the feature rearrangement: $F^0 \to F^2 \to F^1 \to F^3$

3.2.2 Commutative Law

To prove the commutative laws of mixed union and subtraction HBS-Boolean operations, it is necessary to investigate a case of exchanging the j -th operation $\hat{\otimes}_j P_j$ and the $j+1$ -th operation $\hat{\otimes}_{j+1} P_{j+1}$. Let us assume that the original positions of F_j and F_{j+1} are an arbitrary x and y , respectively. Then, the model M_{j+1} at LOD = $j+1$ is represented by the following formula

$$M_{j+1} = M_{j-1} \hat{\otimes}^x_j P^x \hat{\otimes}^y_{j+1} P^y \tag{5}$$

On the other hand, if M'_{j+1} denotes the model after the exchange of the j -th and the $j+1$ -th operations, then it can be represented as follows

$$M'_{j+1} = M_{j-1} \hat{\otimes}^y_j P^y \hat{\otimes}^x_{j+1} P^x \tag{6}$$

To prove that the HBS-Boolean operations satisfy the commutative law, it is necessary to show that Eq. (5) is equal to Eq. (6), i.e. $M_{j+1} = M'_{j+1}$.

According to the algorithm described in Section 3.2.1, M_{j+1} can be represented by $M_{j+1} = \prod_{i=0}^{j+1} {}^{(n-j-1)}\otimes_i {}^{(n-j-1)}P_i$. Let us assume that in Step $n-j-1$ the locations of P^x and P^y are an arbitrary s and t, respectively. If P^y and P^x are moved to the $j+1$-th and j-th place in turn, then M_{j+1} can be represented as follows

$$
\begin{aligned}
M_{j+1} &= \left(\prod_{i=0, i \neq t}^{j} {}^{(n-j-1)}\otimes_i {}^{(n-j-1)}P_i \right) \otimes^y Z_{j+1}^y \\
&= \left(\prod_{i=0, i \neq s, i \neq t}^{j} {}^{(n-j-1)}\otimes_i {}^{(n-j-1)}P_i \right) \otimes^x Z_j^x \otimes^y Z_{j+1}^y \\
&= \left(\prod_{i=0}^{j-1} {}^{(n-j+1)}\otimes_i {}^{(n-j+1)}P_i \right) \otimes^x Z_j^x \otimes^y Z_{j+1}^y \\
&= M_{j-1} \otimes^x Z_j^x \otimes^y Z_{j+1}^y \\
&= M_{j-1} \hat{\otimes}_j^x P^x \hat{\otimes}_{j+1}^y P^y \,.
\end{aligned}
\tag{7}
$$

Next, let us apply the relocation operation in a different order. If P^x and P^y are moved to the $j+1$-th and j-th place in turn, then M_{j+1} can be represented as follows.

$$
\begin{aligned}
M_{j+1} &= \left(\prod_{i=0, i \neq s}^{j} {}^{(n-j-1)}\otimes_i {}^{(n-j-1)}P_i \right) \otimes^x Z_{j+1}^x \\
&= \left(\prod_{i=0, i \neq s, i \neq t}^{j} {}^{(n-j-1)}\otimes_i {}^{(n-j-1)}P_i \right) \otimes^y Z_j^y \otimes^x Z_{j+1}^x \\
&= \left(\prod_{i=0}^{j-1} {}^{(n-j+1)}\otimes_i {}^{(n-j+1)}P_i \right) \otimes^y Z_j^y \otimes^x Z_{j+1}^x \\
&= M_{j-1} \otimes^y Z_j^y \otimes^x Z_{j+1}^x \\
&= M_{j-1} \hat{\otimes}_j^y P^y \hat{\otimes}_{j+1}^x P^x \,.
\end{aligned}
\tag{8}
$$

From Eqs. (6), (7) and (8), the relationship $M_{j+1} = M'_{j+1}$ is established, regardless of the operation type. Therefore, the HBS-Boolean operations satisfy the commutative law for union and subtraction.

4 Implementation

The HBS-Boolean operation was implemented in a feature-based nonmanifold modeling system that was developed based on the Partial Entity Structure. The data structures proposed in Ref. [7], such as multi-resolution features and the merged set, were also used to facilitate the implementation. The Boolean operations were implemented using the merge-and-select algorithm. For a case study, the well-known ANC-101 test part was chosen. Figure 5 shows the modeling process of the ANC-101 part composed of 8 steps. Figure 6 shows the result of the example where the features were rearranged in the order specified by the authors.

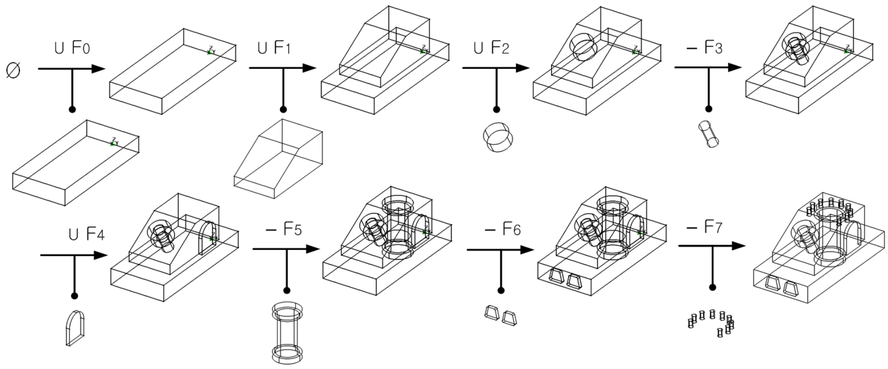

Fig. 5. Feature-based modeling process for the CAM-I ANC-101 test part

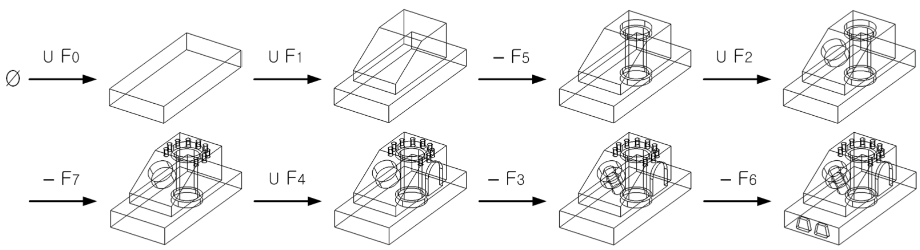

Fig. 6. Multi-resolution models where the features are rearranged in descending order of volume, regardless of the feature type

5 Conclusions

We have proposed HBS-Boolean operations for feature-based multi-resolution modeling, which satisfy the commutative laws for union and subtraction. Because union and subtraction are commutative, then for an arbitrary rearrangement of features, these operations guarantee the same result as the original part shape, and provide reasonable and unique shapes at each intermediate LOD independent of the order of the feature relocation operations. The HBS-Boolean operations were implemented based on a nonmanifold boundary representation and the merge-and-select algorithm. This approach allows for fast extraction of multi-resolution models for given LODs. Although a nonmanifold representation was adopted in this work, as the HBS-Boolean operations are independent from the topological frameworks, they can be implemented using conventional solid data structures, as well as nonmanifold data structures. More applications of the HBS-Boolean operations should be investigated in the future.

Acknowledgement

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