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Feature-based multiresolution techniques for product design^{*}

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Abstract: 3D computer-aided design (CAD) systems based on feature-based solid modelling technique have been widely spread and used for product design. However, when part models associated with features are used in various downstream applications, simplified models in various levels of detail (LODs) are frequently more desirable than the full details of the parts. In particular, the need for feature-based multiresolution representation of a solid model representing an object at multiple LODs in the feature unit is increasing for engineering tasks. One challenge is to generate valid models at various LODs after an arbitrary rearrangement of features using a certain LOD criterion, because composite Boolean operations consisting of union and subtraction are not commutative. The other challenges are to devise proper topological framework for multiresolution representation, to suggest more reasonable LOD criteria, and to extend applications. This paper surveys the recent research on these issues.

Key words: Multiresolution, Level of detail (LOD), Feature, Solid, Non-manifold

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INTRODUCTION

3D CAD systems for the feature-based B-rep solid models (Choi *et al.*, 2002; Kim *et al.*, 2003; 2005; Koo and Lee, 2002; Lee *et al.*, 2004; Lee, 2005a) are becoming more suitable for CAD when compared with the conventional polygon-based multiresolution modelling in computer graphics (Cignoni *et al.*, 1998; Schröder *et al.*, 1992). As illustrated in Fig. 1b, the object of the feature-based multiresolution modelling is a solid model where the suppressed objects are form features at an even higher level of modelling entities than the topological entities.

The applications of feature-based multiresolution modelling are mainly engineering tasks such as analysis, network-based collaborative design, virtual prototyping and manufacture. In engineering analysis,

as shown in Fig. 1, the multiresolution representation of a solid part model provides simplified analysis models at various levels of detail (LODs), as such simplified models are often required rather than the full details of the part (Armstrong, 1994; Lee *et al.*, 2005a; Belaziz and Bouras, 2000). In the distributed design environment, the efficient transmission of solid models over the network is necessary for efficient collaborative design and manufacture (Bidarra *et al.*, 2002; Lee *et al.*, 2004; Li *et al.*, 2003; Wu *et al.*, 2000). Multiresolution representation allows the incremental transmission of solid models and sharing of the model at adequate LOD depending on the engineering tasks to be undertaken. In virtual prototyping and manufacture, LOD techniques are essential for rendering, collision detection, and various engineering analyses and simulations because a digital mockup and a virtual factory contain vast quantities of geometric data.

In recent years, there has been significant research achievement on the feature-based multiresolution modelling technique. The research has focused

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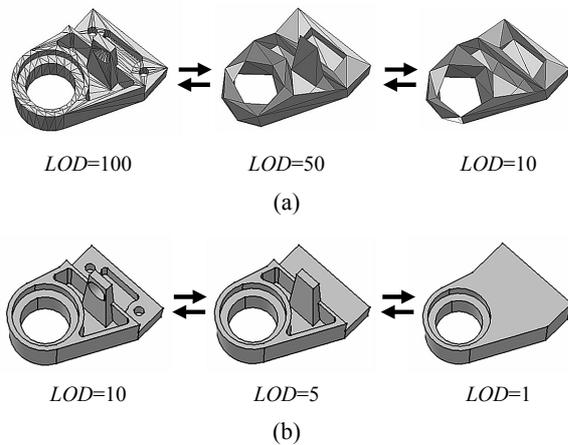


Fig.1 Two different multi-resolution modelling approaches. (a) Polygon-based multi-resolution modelling in computer graphics; (b) Feature-based multi-resolution modelling in CAD

on several topics: topological frameworks for representing multiresolution solid model, criteria for the LOD, generation of valid models after rearrangement of features, and applications. This paper surveys the relevant research on these topics.

TOPOLOGICAL FRAMEWORKS FOR MULTIREOLUTION REPRESENTATION

Two approaches have been identified, using either conventional solid boundary representations (B-rep) or non-manifold B-reps.

Solid B-rep

Choi *et al.* (2002) studied multiresolution modelling for B-rep part models in feature-based solid modelling systems such as SolidWorks. In that paper, features are classified into two groups: additive and subtractive (Dunn, 1992; Lee, 1999; Shah and Mäntylä, 1995). The model at the lowest resolution is constructed by uniting all the additive features, while the models at higher resolutions are generated by applying subtractive features in descending order of volume. To implement this, a hierarchical feature tree for the multiresolution representation is constructed from the original feature history tree. In the hierarchical feature tree, the leaf node at the highest level contains a solid created by uniting all of the additive features, whereas the leaf nodes at the lower levels

contain solids for the remaining subtractive features. The volume of subtractive feature interfering with additive features may need to be redefined, as the union and subtraction operations are not commutative. The model at a certain LOD, the LOD model, is represented by pruning the branches of the feature tree. However, this method is only applicable to the specific feature rearrangement method they suggested. If the features are rearranged in an arbitrary order, the method does not guarantee the same resulting shape as the original solid model.

This method has an advantage because it may be implemented in current commercial 3D CAD systems, as they share the same data structures. However, it requires much computation time to evaluate boundaries, as the transition from the current LOD m to the destination LOD n requires $|m-n|$ Boolean operations (which are the most time-consuming operations used in solid modelling). As the purpose of multiresolution modelling is to rapidly obtain the LOD models, due to the consumption of more data storage, this method is far from ideal.

Kim *et al.* (2003) extended the method of Choi *et al.* (2002) by adding two optional tasks. The first is to simplify the sketches of features. Because the feature made by a complex sketch increases the complexity of a B-rep model, a low-resolution model can be generated by simplifying its sketch. The second task is to remove insignificant additive features. If the volume of a feature is very small compared to that of the original part model, this feature can be ignored in the feature tree. However, there is no discussion of where an additive feature interferes with any subtractive features.

Kim *et al.* (2005) developed three operators for feature-based multiresolution modelling of an assembly, based on Parasolid, a solid modelling kernel. They are: wrap-around, smooth-out, and thinning operator. Through appropriately applying these operators sequentially, an assembly model of any desired resolution can be easily generated. Of course, the assembly can go back to the finer resolution. However, the performance of these multi-resolution operators is heavily dependent on the capability of detecting MR-features. Therefore, if the features of the CAD system can be converted to MR-features by feature mapping, then the performance would be enhanced.

Non-manifold B-rep

1. Cellular model

To solve the problems of the solid-based approach, the non-manifold topological (NMT) model of a cellular structure was introduced as the topological framework of a multiresolution model (Lee *et al.*, 2004; Lee, 2005b). The NMT model can represent any combination of wireframe, surface, solid, and cellular models in a unified data structure, and Boolean operations are closed in the representation domain of NMT models, unlike solid models (Charlesworth and Anderson, 1995; Lee, 1999). Several data structures have been proposed to represent NMT objects (Gursoz *et al.*, 1990; Lee and Lee, 2001; Rossignac and O'Conner, 1990; Weiler, 1998; Yamaguchi and Kimura, 1995).

In the NMT-based method, all features are merged into an NMT cellular model first, and then, if an LOD is given, the topological entities that constitute the model at the required LOD are selected and displayed. Since the boundary information on all of the features is stored in the NMT cellular model, there is no geometric calculation for the boundary evaluation of LOD models. As a result, a model at a given LOD can be provided more quickly than if the solid-based approach is used.

Lee *et al.* (2004) applied the feature-based multiresolution modelling method based on the NMT cellular model to network-based collaborative design. They addressed the incremental transmission of solid models through a network and sharing of the model at adequate LOD for engineering tasks. The ACIS kernel was used to implement the system.

Lee (2005b) adopted the Partial Entity Structure (Lee and Lee, 2001) as the topological framework for multiresolution representation, and introduced the merge and select algorithm for boundary evaluation.

In the merge and select algorithm (Crocker and Reinke, 1991; Masuda, 1993), all primitives are merged into a single boundary representation, a merged set that contains a complete description of the input primitives, all intersections between them, and historical information describing origins of the entities in terms of the topological entities of the original primitives. In our system, all historical information is stored in the cell topological entities. A Boolean logic evaluator, whose input is the CSG representation and the history, makes the selection of merged set entities

corresponding to the Boolean result. The user can modify Boolean operators or their order of occurrence even more easily; he simply re-executes the selection process. He can also perform selection with a CSG tree that contains only a subset of the primitives in the merged set, and selectively filter out primitives from the final result without actually removing them from the merged set.

In the multiresolution modelling method using the merge and select algorithm, all features are first merged into an NMT model, and then, if the LOD is given, topological entities comprising the LOD model are selected and displayed.

Fig.2 shows the merged set composed of five features. The properties of the merged set are very useful for feature-based multiresolution modelling. Once the merged set of all features is generated, any LOD models can be extracted very quickly from the merged set. Therefore, it is a challenging research issue to find a proper set of Boolean operations on the feature volumes to define an LOD model.

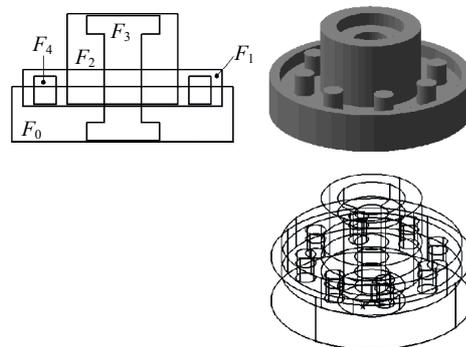


Fig.2 A merged set of the features

2. Mixed dimensional model

Geometric models for analysis are simplified and idealized models that may include not only solid models but also medial surfaces and wireframes. Depending on the engineer's intent and the desired accuracy of analysis results, geometric models at various levels of abstraction (LOA) need to be provided for CAE systems. Extension of the representation domain from solid to nonmanifold models is required to support the multiresolution analysis model in the mixed dimension.

In Lee (2005)'s work, the mixed dimensional NMT model is introduced for multiresolution and

multi-abstraction modelling for seamless CAD-CAE integration. Here, different types of geometric models are simultaneously created for design and analysis for each feature modelling operation. As illustrated in Fig.3, these are merged into a part master model, which is an NMT model called a merged set. Solid models at various LODs can be immediately extracted from the master model. Moreover, for a specific LOD, abstracted NMT models at various LOAs can be rapidly extracted from the master model and transferred to CAE systems. For design changes, modification of the master model results in the simultaneous and consistent modification of the design and analysis models. This CAD/CAE-integrated approach to provide a unified and concurrent modelling environment is a good application of the multiresolution NMT modelling technique.

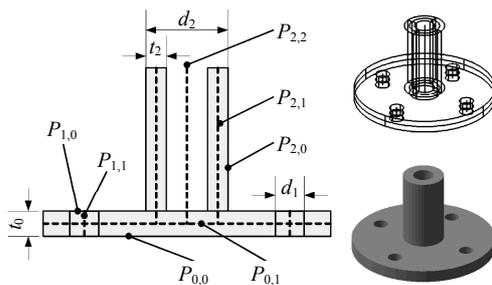


Fig.3 Merged set of the features

VALID LOD MODELS AFTER FEATURE RE-ARRANGEMENT

Feature rearrangement problem

Research has been conducted to generate valid models at various LODs after the feature rearrangement, based on the LOD criterion. In general, if the features are rearranged, the resulting shape is different from the original because union and subtraction Boolean operations are not commutative. For example, let us assume that the example part is created through the feature-based modelling process illustrated in Fig.4. Then, if the features are rearranged to $F_0 \rightarrow F_2 \rightarrow F_1 \rightarrow F_4 \rightarrow F_3$, as shown in Fig.5, the result is different from what it should be.

However, to apply feature-based multiresolution modelling to a wide range of application areas, a final result must be the same as the original shape, and the

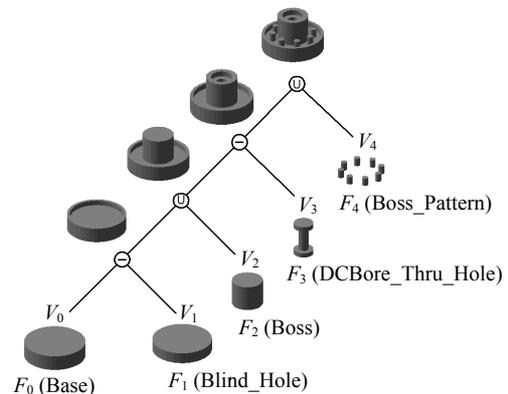


Fig.4 An example of feature-based solid modeling

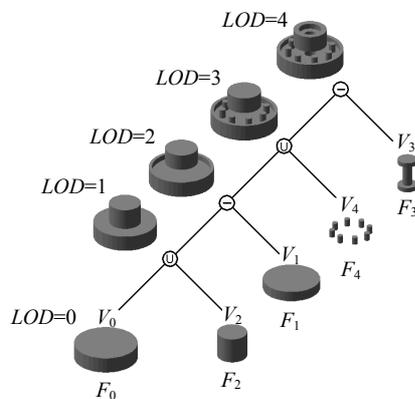


Fig.5 A rearranged feature tree and its results

intermediate LODs of the models must have a reasonable shape, even though the features are rearranged arbitrarily regardless of whether the feature is additive or subtractive. This problem is called the feature rearrangement problem in this area.

Delta volume approach

Choi *et al.*(2002) and Lee *et al.*(2004) proposed algorithms to support a feature arrangement method where additive features take precedence over subtractive features and then subtractive features are rearranged in the descending order of volumes. Here, the model at the lowest resolution is constructed by uniting all of the additive features, while the models at higher resolutions are generated by applying subtractive features in descending order of volume. To implement this, the volume of subtractive feature interfering with additive features may need to be redefined, as the union and subtraction operations are not commutative. This redefined volume is called the

delta volume. The model at a certain LOD, the LOD model, is represented by subtracting the delta volumes from the roughest LOD model.

However, their method is only applicable to the specific feature rearrangement method they suggested. If the features are rearranged in an arbitrary order, their method does not guarantee the same resulting shape as the original solid model. Therefore, it is necessary to find a solution to the arbitrary feature rearrangement problem.

Effective volume approach

To solve the feature rearrangement problem, Lee (2005b) introduced the concept of the effective volume of a feature. When looking into the Boolean operations, we can find that the region influenced by a Boolean operation is altered when the applying order of the operation is changed. This is why union and difference Boolean operations do not follow the commutative law. Thus, in order to obtain the same resultant shape regardless of feature rearrangement, it is necessary to exclude some feature volume from the original one. This adapted volume is called the effective volume of a feature, or the effective feature volume for short.

Let V_i denote the volume of the solid primitive of a feature F_i , \otimes_i denote \cup or $-$ Boolean operation, and M_n denote the resulting model obtained by applying n Boolean operations between $n+1$ solid models:

$$M_n = \prod_{i=0}^n \otimes_i V_i, \text{ where } \otimes_0 V_0 = \emptyset \otimes_0 V_0. \quad (1)$$

If the j th Boolean operation $\otimes_j V_j$ is moved to the m th position, M_n can be represented as follows

$$M_n = \left(\prod_{i=0, i \neq j}^m \otimes_i V_i \right) \left(\otimes_j \left(V_j - \sum_{l=1}^{m-j} \varphi(\otimes_j, \otimes_{j+l}) V_{j+l} \right) \right) \cdot \left(\prod_{i=m+1}^n \otimes_i V_i \right), \quad (2)$$

where

$$\varphi(a, b) = \begin{cases} 0, & \text{if } a = b; \\ 1, & \text{if } a \neq b. \end{cases}$$

This method guarantees the same resulting shape for an arbitrary rearrangement of the features, re-

gardless of whether feature types are additive or subtractive. This characteristic enables various LOD criteria to be used for a wide range of applications. In addition, the effective volume of a feature is independent of the data structure of the model. Therefore, the multiresolution modelling algorithm can be implemented using the conventional B-rep solid representation as well as the NMT representation.

History-based selective Boolean operations

For an arbitrary feature rearrangement, the feature relocation operation using Eq.(2) guarantees the same resulting shape as the original. However, the intermediate LOD models are altered, depending on the order of the feature relocation operations. Moreover, there is no criterion to decide which is the most reasonable shape among the various options.

The reason for this is that at each operation, the effective volume of the moved feature is calculated using the current definition of the effective volume. Therefore, to form intermediate LOD models with an acceptable shape using the operation based on Eq.(2), this order must be selected carefully.

To solve this problem, Lee et al.(2006) proposed history-based Boolean (HBS-Boolean) operations that obey commutative laws for union, subtraction, and intersection operations.

In the HBS-Boolean operation, the volume of each feature is refined considering two reasons why the resulting shape of the reordered Boolean operations is different from the original: (1) mixed union and subtraction operations do not obey the commutative law, and (2) Boolean operations are always applied to the entire shape, and thus, the region affected by each Boolean operation in the initial creation order is different from the affected region in the rearranged order.

The refinement is conducted by excluding the overlapping volume of the feature that satisfies the following conditions from the volume of each feature: (1) the feature located at the post-position in the initial creation order, but at the pre-position in the rearranged order, and (2) the feature of a different feature type (additive or subtractive).

The definition of the HBS-Boolean operations can be formalized from the refinement method above. Let F_j^i denote a feature that is applied at the i th place in the original order, but is now located at the j th place

in the rearranged order. If $\hat{\otimes}$ denotes an HBS-Boolean operation, then the corresponding HBS-Boolean operation of F_j^i is $\hat{\otimes}_j^i P^i$. If Z_j^i denotes the refined volume of F_j^i , then the HBS-Boolean operation $\hat{\otimes}_j^i P^i$ can be represented by Eq.(3).

HBS-Boolean operation:

$$\hat{\otimes}_j^i P^i = \otimes^i Z_j^i, \tag{3}$$

where

$$Z_j^i = P^i - \sum_{l=0}^{j-1} \varphi(j,l) \gamma(i,k(l)) P_l^{k(l)}, \tag{4}$$

where

$$\varphi(i,j) = \begin{cases} 1, & \text{if } \otimes_i \neq \otimes_j; \\ 0, & \text{otherwise} \end{cases}, \quad \gamma(i,j) = \begin{cases} 1, & \text{if } i < j; \\ 0, & \text{otherwise} \end{cases},$$

and $k(l)$ is the initial location (in the original order) of the current l th feature (in the rearranged order).

The HBS-Boolean operations obey the commutative law between different types of HBS-Boolean operations. By virtue of the commutative property, for an arbitrary feature rearrangement, these operations guarantee the same resulting shape as the original, and also a unique and acceptable shape at each intermediate LOD independent of the order of the feature relocations. Actually, the result of the HBS-Boolean operations is the same as that of the feature rearrangement algorithm that relocates the features in turn from the least significant feature in proportion to the feature significance.

CRITERIA FOR THE LOD

Volume of subtractive feature

The criteria of LOD are application-dependent. One possible LOD criterion is the volume of the feature, which has been proposed by Choi *et al.*(2002) and Lee *et al.*(2004). In this approach, additive features take precedence over all subtractive features, and then subtractive features are rearranged in descending order of their volume sizes. The lowest resolution model is a volume enclosing all additive features, and the higher resolution models are obtained by removing the volumes of subtractive features from the enclosing volume in the descending

order. This method is used for applications such as rendering and streaming solid models.

If this criterion is applied to the example in Fig.4, the features are rearranged as $F_0 \rightarrow F_2 \rightarrow F_4 \rightarrow F_1 \rightarrow F_3$. Since three features F_0 , F_2 , and F_4 are of the additive type and the lowest LOD model is their union set. As a result, three different LOD models shown in Fig.6 can be extracted from this multiresolution model.

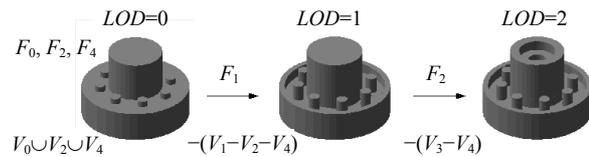


Fig.6 Three LOD models for the reordered features according to the subtractive volume

In this method, the LOD criterion is the whole volume of the subtractive feature. However, the whole volume of the subtractive feature may not be used to subtract the feature from the part model. Only the volume $V_i \cap (M_0 - M_n)$ contributes to the Boolean result. Here, V_i denotes the primitive volume of the feature F_i , and M_i is the LOD model at $LOD=i$. Thus, this intersection volume may be a more reasonable LOD criterion than the whole feature volume.

Moreover, this method has a few more serious problems as a general LOD criterion. First, some additive features can be more detailed than subtractive features. It means that the lowest resolution model may have even more detailed shape than the higher resolution models. Second, if a part is modelled using only additive features, any LOD model cannot be offered as there is only one LOD. To overcome these problems, the volume size of feature was suggested by Lee (2005b) as a new criterion. This criterion is more general, and works regardless of whether the feature is additive or subtractive.

Volume of feature

The criterion of the volume size of feature can be branched into a few more detailed criteria according to which volume is measured. First of all, the simplest one is the volume of the feature itself. However, as mentioned above, this may include the volume that does not contribute to the final resulting shape.

To solve this problem, Lee (2005b) proposed the method as follows. First, the least significant feature

and mid-surface models of the main part shape as a base feature. Next, the user creates sub-features sequentially. All geometric models of each sub-feature for design and analysis are merged into the part master model.

In the idealization phase, the feature-based idealization module is executed. If the user specifies the LOD, then the corresponding solid model is extracted from the master model. A series of solid models can be extracted at a sequence of LODs. These multiresolution models can be used for analysis systems that require CAD solid models as geometric input. If the analysis system requires abstract models, then the multiresolution models for various LODs and LOAs can be extracted from the master model. The analysis model is chosen from these multiresolution models, and then transferred to the analysis system.

In the analysis phase, various analyses such as structural analysis and molding process simulation are conducted in this phase. If the analysis results are not satisfactory, the user goes to the design or the analysis stage. This design-analysis cycle is iterated until the analysis results are satisfactory.

Fig.9 presents the idealization results of the example part model for varying LOD and LOA.

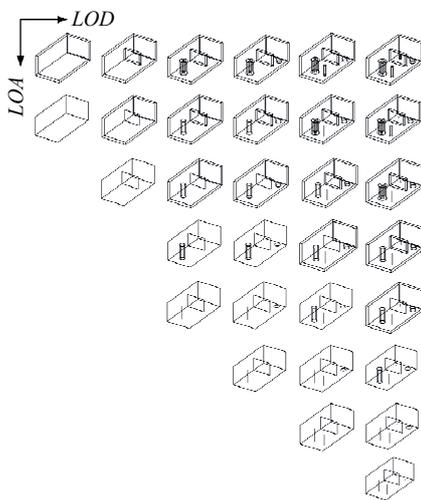


Fig.9 Idealized models using feature-based multiresolution and multi-abstraction techniques

CONCLUSION

This paper surveys recent research on the feature-based multiresolution modelling technology for solid and NMT CAD models. The technology can be

used in a wide range of applications that are not only computer graphics but also engineering tasks such as analysis. Recently, there has been significant research achievement on this technology. However, there still remains a lot of future work to complete this technology as follows.

(1) Fast multiresolution modelling technique for practical use. The effort to accomplish quick response has been concentrated so far. However, current approaches still require a lot of computation time for a large CAD model. As the users in front of CAD systems do not tolerate late response of multiresolution modeler, it is necessary to develop a faster multiresolution modelling technique for a large CAD model.

(2) Simultaneous multiresolution modelling capability. The current approaches support only the multiresolution representation of a completed design model. If the multiresolution models can be provided immediately at anytime during the design process, it would be very helpful for product design.

(3) New applications and LOD criteria. The multiresolution and multi-abstraction modelling techniques may be applicable to a wider range of application areas that are not only engineering analysis but also diverse applications such as visualization, network-based collaborative design, and virtual prototyping and manufacturing. In addition, more LOD criteria may need to be suggested for more applications, as LOD criterion is usually application-dependent.

(4) Feature-based multiresolution modelling of assembly models. This is useful for virtual prototyping and manufacturing in particular. Although there are a few articles related with this topic, it is still necessary to investigate a new method. Assembly constraints should be considered when generating multiresolution assembly models.

(5) More robust and seamless integration for CAD and CAE. The work includes integration of the MAT approach to overcome a limitation of the design-by-feature method, development of a more robust embedding method of an analysis model into the part model, and investigation of the proper transfer method of boundary conditions specified on the CAD model to the CAE model.

References

- Armstrong, C.G., 1994. Modelling requirements for finite element analysis. *Computer-Aided Design*, **26**(7):573-578.

- [doi:10.1016/0010-4485(94)90088-4]
- Belaziz, M., Bouras, A., Brun, J.M., 2000. Morphological analysis for product design. *Computer-Aided Design*, **32**(5-6):377-388. [doi:10.1016/S0010-4485(00)00019-1]
- Bidarra, R., Kranendonk, N., Noort, A., Bronsvooort, W.F., 2002. A Collaborative Framework for Integrated Part and Assembly Modelling. Proceedings of the 7th ACM Symposium on Solid Modelling and Applications. Saarbrücken, Germany, p.389-400.
- Charlesworth, W.W., Anderson, D.C., 1995. Applications of Non-manifold Topology. Proc. of International Computers in Engineering Conference and the ASME Database Symposium, p.103-112.
- Choi, D.H., Kim, T.W., Lee, K., 2002. Multiresolutional representation of b-rep model using feature conversion. *Transactions of the Society of CAD/CAM Engineers*, **7**(2):121-130 (in Korean)
- Cignoni, P., Montani, C., Scopigno, R., 1998. A comparison of mesh simplification algorithms. *Computers & Graphics*, **22**(1):37-54. [doi:10.1016/S0097-8493(97)00082-4]
- Crocker, G.A., Reinke, W.F., 1991. An editable non-manifold boundary representation. *IEEE Computer Graphics & Applications*, **11**(2):39-51. [doi:10.1109/38.75589]
- Dunn, M., 1992. Industrial Automation Systems and Integration—Product Data Representation and Exchange—Part 48: Integration Generic Resources: Form Features, Second Ed. ISO/WD 10303-48.
- Gordon, S., 2001. An Analyst's View: STEP-enabled CAD-CAE Integration. Presentation Materials of NASA's STEP for Aerospace Workshop, p.16-19.
- Gursoz, E.L., Choi, Y., Prinz, F.B., 1990. Vertex-based Boundary Representation of Non-manifold Boundaries. In: Wozny, M.J., Turner, J.U., Preiss, K. (Eds.), *Geometric Modelling for Product Engineering*. North-Holland, p.107-130.
- Kim, S.C., Lee, K., Kim, T.W., 2003. Feature Tree Pruning Operation to Generate Multi-resolution Model. Proceedings of 2003 Conference of the Society of CAD/CAM Engineers, Seoul, Korea, p.397-402 (in Korean).
- Kim, S.C., Lee, K., Hong, T., Kim, M., Jing, M., 2005. An Integrated Approach to Realize Multi-resolution of B-rep Model. Proceedings of the 2005 ACM Symposium on Solid and Physical Modelling, p.153-162.
- Koo, S., Lee, K., 2002. Wrap-around operation to make multiresolution model of part and assembly. *Computers & Graphics*, **26**(5):687-700. [doi:10.1016/S0097-8493(02)00124-3]
- Lee, J.Y., Lee, J.H., Kim, H., Kim, H.S., 2004. A cellular topology-based approach to generating progressive solid models from feature-centric models. *Computer-Aided Design*, **36**(3):217-229. [doi:10.1016/S0010-4485(03)00094-0]
- Lee, K., 1999. Principles of CAD/CAM/CAE Systems. Addison Wesley Longman, Inc.
- Lee, K.Y., Armstrong, C.G., Price, M.A., Lamont, J.H., 2005a. A Small Feature Suppression/Unsuppression System for Preparing B-rep Models for Analysis. Proceedings of the 9th ACM Symposium on Solid and Physical Modelling. Cambridge, Massachusetts, UK, p.113-124.
- Lee, S.H., 2005a. Feature-based multiresolution modelling of solids. *ACM Trans. on Graphics*, **24**(4):1417-1441. [doi:10.1145/1095878.1095887]
- Lee, S.H., 2005b. A CAD-CAE integration approach using feature-based multi-resolution and multi-abstraction modelling techniques. *Computer-Aided Design*, **37**(9):941-955. [doi:10.1016/j.cad.2004.09.021]
- Lee, S.H., Lee, K., 2001. Partial entity structure: A compact boundary representation for non-manifold geometric modelling. *Journal of Computing and Information Science in Engineering*, **1**(4):356-365. [doi:10.1115/1.1433486]
- Lee, S.H., Lee, K., Lee, K.Y., 2005b. Feature-based Multiresolution and Multi-abstraction Non-manifold Modelling System to Provide Integrated Environment for Design and Analysis of Injection Molding Products. Proceedings of the First Korea-China Joint Conference on Geometric and Visual Computing, p.179-187.
- Lee, S.H., Lee, K., Kim, S.C., 2006. History-based Selective Boolean Operations for Feature-based Multi-resolution Modelling. Proceedings of ICCSA 2006, LNCS **3980**:459-468.
- Li, J., Gao, S., Zhou, X., 2003. Direct Incremental Transmission of Boundary Representation. Proceedings of the 8th ACM Symposium on Solid Modelling and Applications. Seattle, Washington, USA, p.298-303.
- Masuda, H., 1993. Topological operators and Boolean operations for complex-based nonmanifold geometric models. *Computer-Aided Design*, **25**(2):119-129. [doi:10.1016/0010-4485(93)90097-8]
- Rossignac, J., O'Conner, M.A., 1990. SGC: A Dimensional-independent Model for Pointsets with Internal Structures and Incomplete Boundaries. In: Wozny, M.J., Turner, J.U., Preiss, K. (Eds.), *Geometric Modelling for Product Engineering*. North-Holland, p.145-180.
- Schröder, W.J., Zerge, J.A., Lorensen, W.E., 1992. Decimation of triangle meshes. *ACM SIGGRAPH Computer Graphics*, **26**(2):65-70. [doi:10.1145/142920.134010]
- Shah, J.J., Mäntylä, M., 1995. Parametric and Feature-based CAD/CAM. John Wiley & Sons, Inc.
- Weiler, K., 1998. The Radial Edge Structure: A Topological Representation for Non-manifold Geometric Boundary Modelling. In: Wozny, M.J., McLaughlin, H.W., Encarnação, J.L. (Eds.), *Geometric Modelling for CAD Applications*. North-Holland, p.3-36.
- Wu, D., Dhargava, S., Sarma, R., 2000. Solid Model Streaming as the Basis for a Distributed Design Environment. Proceedings of the 2000 ASME Design Engineering Technical Conferences & Computers and Information in Engineering Conference, 26th Design Automation Conference. Baltimore, Maryland, DETC2000/DAC-14250.
- Yamaguchi, Y., Kimura, F., 1995. Nonmanifold topology based on coupling entities. *IEEE Computer Graphics and Applications*, **15**(1):42-50. [doi:10.1109/38.364963]